Worksheet #17: Some simple PDE's

(1) Find a general solution to $tu_{xx} - 4u_x = 0$. [Hint: Try a substitution.]

let $v = u_x'$

$t v_x' - 4v = 0 \rightarrow v_x - \frac{4}{t} v = 0$ linear use integrating factor $\mu = e^{-4x/t}$

$(e^{-4x/t} v)' = 0 \rightarrow v = Ce^{4x/t}$

$\rightarrow u'(x) = Ce^{4x/t} \rightarrow u(x) = \frac{Ct}{4} e^{4x/t} + D$.

(2) (a) Find a general solution to $u_{xt} + \dfrac{u_t}{x} = \dfrac{t}{x^2}$. [Hint: Try integrating with respect to t first.]

integrate wrt t

$\frac{d}{dt}\left(u_x + \frac{u}{x}\right) = \frac{t}{x^2} \rightarrow u_x + \frac{u}{x} = \frac{t^2}{2x^2}$ now use integrating factor $\mu = e^{\int 1/x} = x$

$(xu)' = \frac{t^2}{2x} \rightarrow xu = \frac{t^2}{2} \ln x + C \rightarrow u(x) = \frac{t^2}{2x} \ln x + \frac{C}{x}$.

(b) Check that your solution satisfies the original PDE.

$u_x = \frac{t^2}{2}\left(\frac{1}{x^2} - \frac{1}{x^2}\ln x\right) - \frac{C}{x^2}$ $u_t = \frac{2t}{2x} \ln x$ $u_{xt} = t\left(\frac{1}{x^2} - \frac{1}{x^2}\ln x\right)$

$u_{xt} + \frac{u_t}{x} = \frac{t}{x^2} - \frac{t}{x^2}\ln x + \frac{t}{x^2}\ln x = \frac{t}{x^2}$ ✓

(b) Can you integrate with respect to x first?

Yes let $V = u_t$

$V_x + \frac{V}{x} = \frac{t}{x^2}$ use an integrating factor

(c) If you want to solve an ODE in x first, try it. Do you get the same answer?

Yes you will get the same answer.