Worksheet #16: Green's functions

Consider the differential operator

$$A = -\frac{d^2}{dx^2}$$

on $[0, 1]$ with Dirichlet boundary conditions. We wish to find a Green's function for problems of the form

$$Au = f$$
$$u(0) = u(1) = 0.$$

In this worksheet, we will derive the Green's function.

- Write the general solution to $Au = 0$.

- Solve for $u_1(x)$ which only satisfies the left-hand boundary conditions.

- Solve for $u_2(x)$ which only satisfies the right-hand boundary conditions.

- Compute the Wronskian. ie $W = \det\left(\begin{bmatrix} u_1(x) & u_2(x) \\ u_1'(x) & u_2'(x) \end{bmatrix}\right)$

- Write $g(x, \xi)$.

- Sketch $g(x, \xi)$ in the box $[0, 1]^2$. Do you notice anything?