

Worksheet #12: Real or not... The story of Sturm-Liouville eigenvalues

Consider the Sturm-Liouville problem with $p = 1$ and $q(x)$ real:

$$-y'' + q(x)y = \lambda y, \quad a < x < b$$

with Dirichlet boundary conditions $y(a) = y(b) = 0$.

(a) Multiply the ODE by \bar{y} .

(b) Multiply the conjugate of the ODE by y .

(c) Subtract the two equations. (There should be some cancellation.)

(d) Integrate over the interval (a, b) . [Hint: use integration by parts]

(e) Apply boundary conditions.

(f) What is the sign of $\int_a^b y\bar{y}dx$. [Hint: If a is a complex number (ie. $a = b + ci$ for b and c real constants) then $a\bar{a} = (b + ci)(b - ci)$]

(g) Conclude something about $\lambda - \bar{\lambda}$. What does this mean about λ ?

(h) What other boundary conditions would this work for? Neumann? Periodic? ($y(a) = y(b)$ and $y'(a) = y'(b)$) Mixed ($y'(a) = \alpha y(a)$ and $y'(b) = \beta y(b)$)