Math 46: Homework 9
Due May 29

(1) Page 372 # 5. This should be easy if you look up the radial part of the 3D Laplacian operator.

(2) Page 372 # 6. Adapt the method from 1D. In fact $-\Delta$ is a positive operator. Note the λ values would be eigenvalues of the Laplacian.

(3) Page 396 # 4. As a function of ξ this is called a Cauchy distribution. It comes up in statistics and has an infinite variance.

(4) Page 396 # 5 b, c. This should be quick. These show that translation becomes multiplication in Fourier space.

(5) Page 396 # 7. Once (or even before!) you have solved, answer this: how is the solution $u(x,t)$ at time t related to the solution for the case $c = 0$ at the same time t ? [Hint: the previous question is useful here]

(6) Use the *sifting property*

$$\int_{-\infty}^{\infty} \delta(x-a)f(x)dx = f(a)$$

to find the Fourier transform of the delta distribution $\delta(x-a)$. Now write the inversion formulathis gives you a new and useful representation of the delta distribution. By interchanging the labels x and ξ, deduce the Fourier transform of the plane wave function e^{ikx}. Add your answer to Table 6.2.

(7) Page 396 # 10. [Hint: write out $|\hat{u}(\xi)| = \hat{u}(\xi)\overline{\hat{u}(\xi)}$ using a double integral, use the above, then simplify]. This is the continuous analogue of Parsevals equality on p. 213. The Fourier transform is a (continuous rather than countably infinite) orthogonal expansion.

(8) Page 397 # 11.

(9) Page 398 # 15. I suggest that you not use the hint until you have a convolution expression for $u(x,y)$ as in Example 6.35, of which you may piggyback off the final result. You may use the boundary condition $\lim_{y\to\infty} u(x,y)$ is bounded. The problem corresponds to injecting current density into the edge of a resisitive medium and solving for the voltage fielda useful medical imaging technique (Electrical Impedance Tomography).

(10) **[Bonus 2 points]** Page 382 # 3 b. Use the result from a, which states that the L given is self-adjoint when certain BCs are imposed. [Hint: see proof we did for Fredholm operators (or, even, symmetric matrices), and it should not be hard].