

Math 46 Homework 6
Due May 8 at the beginning of class

The problems are assigned in an order that will make them easier to solve. (ie. they build on each other.)

Sturm-Liouville problems

- (1) Page 225 # 7.
- (2) Page 225 # 8.

Volterra equations, transferring back and forth to IVPs

- (3) Page 245 # 9. [Hint: Bring out e^t and turn it into an ODE which you should state. Solve the ODE with initial conditions.]
- (4) Page 244 # 6. [You will need to use Leibniz formula from # 1. Do not forget about the initial conditions.]
- (5) Page 246 # 24. [Hint: Use Lemma 4.9] Make sure to state f and K .
- (6) Page 244 # 8. Be careful about using integration variables that are distinct from the limits. You should end up with a polynomial in t .

Fredholm equations with degenerate kernels, equivalent matrix problems

- (7) Page 245 # 13 a,c. a) The simplest possible degenerate kernel, but you could just integrate to solve. State the special value of λ and for this value give condition(s) on f such that a solution exist. c) Now you will have to write down the functions $\alpha_1(x)$, and $\beta_1(x)$.
- (8) Page 244 #3
- (9) Find the spectrum and eigenfunctions of the integral operator

$$(Ku)(x) = \int_0^1 (1 - 5x^2y^2)u(y)dy.$$

Is $Ku - u = f$ solvable given the function $f(x) = x$? If so, find the solution $u(x)$. Is it unique?

Fredholm with continuous kernel

- (10) Page 244 # 4 c. [Hint: Write out the eigenvalue problem and take derivatives until you get a Sturm-Liouville problem.]