

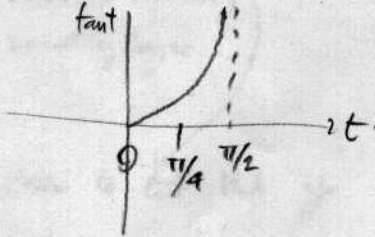
B) Find the scaling of  $x$  with  $\epsilon$  that makes two terms of equal order and others of lower order, in:

$$\epsilon x^4 + \epsilon x^3 - x^2 + 2x - 1 = 0$$

Find the leading-order term in each of the four roots:

If time, continue to higher corrections for these roots:

A) Is  $f(t, \epsilon) = \epsilon \tan t$  uniformly convergent to zero on  $(0, \pi/4)$ ?



yes since  $|\tan t| \leq \text{const}$  on  $(0, \pi/4)$ .

$(0, \pi/2)$ ?  
no since  $\tan t$  unbounded on  $(0, \pi/2)$ .

does  $\epsilon \tan t$  converge pointwise\* on  $(0, \pi/2)$ ?

yes since for any  $t \in (0, \pi/2)$ ,  $\tan t$  is some number  $C$  and  $f(t, \epsilon) = C\epsilon \rightarrow 0$  as  $\epsilon \rightarrow 0$ .

B) Find the scaling of  $x$  with  $\epsilon$  that makes two terms of equal order and others of lower order in

$$\epsilon x^4 + \epsilon x^3 - x^2 + 2x - 1 = 0$$

guess  $x = \epsilon^{-1}$ ?

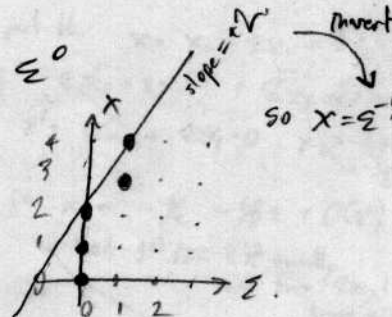
$$\epsilon^{-3} \quad \epsilon^{-2} \quad \epsilon^{-2} \quad \epsilon^{-1} \quad \epsilon^0$$

dominant

$x = \epsilon^{-1/2}$ ?

$$\epsilon^{-1} \quad \epsilon^{-1/2} \quad \epsilon^{-1} \quad \epsilon^{-1/2} \quad \epsilon^0$$

dominantly balanced



Graphical way to solve: find the line connecting 2 points in the (power of  $\epsilon$ , power of  $x$ ) plane with all other points to the right (higher  $\epsilon$  powers).

Find the leading-order term in each of the four roots:

regular roots (drop 1st two terms)  $-x_0^2 + 2x_0 - 1 = 0$  so  $x_0 = \pm 1$  (twice)

sub.  $x = \frac{y}{\epsilon^{1/2}} \Rightarrow \epsilon \frac{y^4}{\epsilon^2} + \epsilon \frac{y^3}{\epsilon^{3/2}} - \frac{y^2}{\epsilon} + 2 \frac{y}{\epsilon^{1/2}} - 1 = 0$

$$\Rightarrow y^4 + \epsilon^{1/2} y^3 - y^2 + 2\epsilon^{1/2} y - \epsilon = 0$$

$$y_0^4 - y_0^2 = 0 \text{ want}$$

If true, continue to higher corrections for these roots:  
tricky: will do more later

ie  $y_0 = 0$  twice,  $\pm 1$

$$x = +1 + \frac{1}{4}\epsilon^{1/2} + O(\epsilon) \text{ (twice)}, \quad \pm \epsilon^{-1/2} - \frac{3}{2} + O(\epsilon^{1/2}) \dots \quad \times = \pm \epsilon^{-1/2}$$