5/16/07
Barnett

MATH 46 WORKSHEET : simple PDEs

- Find a general solution to $t u_{xx} - 4 u_x = 0$ [Hint: compare to lecture example]

- Find a general solution to $u_{xt} + \frac{u_t}{x} = \frac{t}{x^2}$

[Hint: try $\int dt$ first].

Check it satisfies original PDE :

Now try $\int dx$ first instead : — can it be done?

If you want to solve ODE in x first, try it :

Same as before?

5/16/07
Bennett

MATH 46 WORKSHEET : simple PDEs

~ SOLUTIONS ~

* Find a general solution to $t u_{xx} - 4 u_x = 0$ [Hint: compare to lecture example]

set $u_x = v$

then $t v_x - 4v = 0$ $\quad v_x = \left(+\frac{4}{t}\right) v$ const wrt x

$\Rightarrow v(x,t) = c(t) e^{\frac{4}{t}x}$

$\Rightarrow u(x,t) = \int v(x,t)\,dx = \boxed{\frac{t}{4} c(t) e^{\frac{4}{t}x}} + d(x)$

can call $f(t)$, more simply.

* Find a general solution to $u_{xt} + \frac{u_t}{x} = \frac{t}{x^2}$

[Hint: try $\int dt$ first].

$\int dt$ ↓

solve as ODE wrt x ←

$u_x + \frac{u}{x} = \frac{t^2}{2x^2} + f(x)$

integrating fac. is $e^{\int \frac{1}{x} dx} = e^{\ln x} = x$.

$(xu)_x = \frac{t^2}{2x} + x f(x)$

$\Rightarrow xu = \frac{t^2}{2} \ln x + \int x f(x)\,dx + g(t)$

$\Rightarrow u(x,t) = \frac{t^2}{2x} \ln x + F(x) + \frac{g(t)}{x}$.

Check it satisfies original PDE: compute u_t, u_{tx} and sub. in, works.

Now try $\int dx$ first instead — can it be done? yes, use $v = u_t$.

If you want to solve ODE in x first, try it:

Same as before? yes.