Practise Questions: MIDTERM 2 MATH 46 Barnett

1) Find the first 2 terms of the Neumann series for solution of

$$\int_0^t \frac{u(s)}{\sqrt{t-s}}\,ds \;-\; 5u(t) \;=\; t$$

Write an integral giving the 3^{rd} term.

2) Write $\quad tu - u' = \sin t, \quad t>0, \quad u(0)=0$

as a Volterra integral eqn. What is $k(t,s)$ the kernel?

3) Find the Greens function for $\quad -u'' = f \quad , \quad \begin{array}{l} u(0)=0 \\ u'(1)=0 \end{array}$

← note mixed BCs.

Also write it as an eigenfunction expansion.

(compare #4b p. 244).

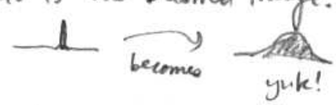
3½) Define the concept of <u>completeness</u> for an orthonormal set in $L^2[a,b]$.

4) Let K be operator in #4c p. 244. Solve $Ku - \frac{\pi}{9}u = \cos 3x$

(possibly corrected #7d!). Discuss existence, uniqueness.

5) An imaging system blurs the (periodic) image $u(x)$ on $[0,2\pi]$ according to

$$Ku(x) = \int_0^{2\pi} [\cos(x-y) + 1]\, u(y)\, dy. \qquad \leftarrow \text{this is the blurred image.}$$



What are eigenfunctions & eigenvalues of K operator? ← try to give complete list.

For 1^{st}-kind equation $Ku = f$ ← some detected image, discuss solvability. Find Ran(K); Is it a good imaging system?

If $f(x) = \sin 2x$ what's the solution? If $f = \sqrt{2}\sin(x+\frac{\pi}{4})$ what's solution?

6) Put $(1-x^2)u'' - xu' + \alpha^2 u = 0$ (Chebyshev's Eqn)

into Sturm-Liouville form

7) Given a continuous 2π-periodic function $g(t)$

show $Ku(t) = \int_0^{2\pi} g(t-s)\, u(s)\, ds$ has

eigenfunctions $u_n(t) = e^{int}$ for $n = \cdots -1, 0, +1, \cdots$

& find eigenvalues λ_n.

Solve the "deconvolution problem" $Ku = f$, ie give closed form expression for $u(x)$.

8) Use Cauchy-Schwarz inequality to put an upper bound on

$\int_0^1 s\, u(s)\, ds$ in terms of $\|u\|$ in $L^2[0,1]$.

9) Find Fourier sine series for $f(x) = \begin{cases} 1 & 0 \le x < \pi/2 \\ 0 & \pi/2 \le x < \pi \end{cases}$ on $[0,\pi]$

Do you expect $\begin{cases} \text{pointwise} \\ \text{uniform} \end{cases}$ convergence? convergence in L^2?

Sketch the resulting function on all of $\mathbb{R}$.

Put an upper bound on the sum of the squared coefficients.

10) Construct an orthonormal set from linear combinations of $\{1, x, x^2\}$ on $[0,1]$.

11) p. 226 # 10.

12) p. 246 #17. & solve it if possible. Also give spectrum & eigenfunctions

13) p. 258 #8 use split function way.