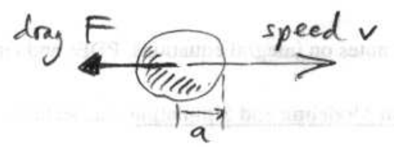
MATH 46 WORKSHEET: Dimensional Analysis. W 3/26/08

Say we suspect that drag force $F$ depends only on a sphere's radius $a$, its speed $v$, and the surrounding fluid density $\rho$.

$$\begin{array}{c} \\ M \\ L \\ T \end{array} \begin{array}{c} \begin{array}{cccc} a & v & \rho & F \end{array} \\ \left[ \begin{array}{cccc} \phantom{a} & \phantom{v} & \phantom{\rho} & \phantom{F} \\ \phantom{a} & & & \\ \phantom{a} & & & \end{array} \right] \end{array}$$




a) Fill in the matrix with the dimensions. [If you want to work out dimensions of $F$, ask your neighbor about Newton's 2nd Law!]

b) Find a dimensionless combination of $a, v, \rho, F$. Call it $\pi = \ldots$

c) Give the vector of powers, ie $\vec{\alpha} = [\alpha_1\ \alpha_2\ \alpha_3\ \alpha_4]$ such that $\pi = a^{\alpha_1} v^{\alpha_2} \rho^{\alpha_3} F^{\alpha_4}$:

$\vec{\alpha} = [\qquad\qquad\qquad]$

Is this choice unique?
What is the form of all such vectors?
What subspace of the 3x4 matrix do they lie in?

d) Can there be another dimensionless combo. (linearly indep. of $\vec{\alpha}$)?
Use linear algebra to prove your claim [Hint: uses matrix rank]

e) So what does Buckingham Pi Theorem tell you?
How must $F$ depend on $a, v, \rho$? $F = \ldots$

sorry, actually inertial drag
← this is ~~Stokes' law~~

f) If $F$ also depended on viscosity $\eta$ (units $ML^{-1}T^{-1}$) repeat part e). [use the back].

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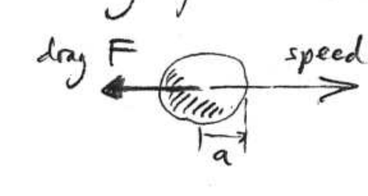
Say we suspect that drag force F depends only on a sphere's radius a, its speed v, and the surrounding fluid density $\rho$.

$$\begin{array}{c|cccc} & a & v & \rho & F \\ \hline M & & & 1 & 1 \\ L & 1 & 1 & -3 & 1 \\ T & & -1 & & -2 \end{array}$$




the key is to recognize you're searching for lin. combo's of columns which gives $\begin{bmatrix}0\\0\\0\end{bmatrix}$, ie $A\vec{\alpha} = \vec{0}$

"F = ma" = $MLT^{-2}$

a) Fill in the matrix with the dimensions. [If you want to work out dimensions of F, ask your neighbor about Newton's 2nd Law!]

b) Find a dimensionless combination of $a, v, \rho, F$. Call it $\Pi = \ldots$

$$\frac{F}{v^2 \rho a^2} \quad \text{or} \quad \frac{v^2 \rho a^2}{F} \quad \text{or} \quad \left(\frac{v^2 \rho a^2}{F}\right)^k \text{ for any } k$$

for this case

c) Give the vector of powers, ie $\vec{\alpha} = [\alpha_1\ \alpha_2\ \alpha_3\ \alpha_4]$ such that $\Pi = a^{\alpha_1} v^{\alpha_2} \rho^{\alpha_3} F^{\alpha_4}$:

$$\vec{\alpha} = [-2\ -2\ -1\ \ 1]$$

Is this choice unique? no (see 2 examples in b).

What is the form of all such vectors? $k\vec{\alpha}$ where $k \in \mathbb{R}$ is scalar.

What subspace of the 3x4 matrix do they lie in? Nul A

Note $\vec{\alpha}$ really is a column vector in $\mathbb{R}^4$. Nullspace $A = \{\vec{\alpha} : A\vec{\alpha} = \vec{0}\}$

d) Can there be another dimensionless combo (linearly indep. of $\vec{\alpha}$)? Use linear algebra to prove your claim [Hint: use matrix rank]

rank A = 3 since there's 3 pivots when row-reduce. dim Nul A = m − rank A = 4 − 3 = 1

$\Rightarrow$ no more lin. indep. $\vec{\alpha}$'s.

e) So what does Buckingham Pi Theorem tell you? Tells you $\Pi_1$ = const.

How must F depend on $a, v, \rho$? $F = c\rho a^2 v^2$ ← inertial drag (low viscosity limit)

unknown const.

this is harder, but get $F = c\rho a^2 v^2 g(\Pi_2)$ where $\Pi_2 = \frac{\eta}{\rho v a}$ = "Reynolds #". ← this is ~~Stokes' law~~

f) If F also depended on viscosity $\eta$ (units $ML^{-1}T^{-1}$) repeat part e) [use the back]