Math 46: Applied Math: Homework 5

due Wed Apr 30 . . . but best if do relevant questions after each lecture

Slightly shorter as last week, for Midterm 1 recovery; some of Fourier stuff is recap of Math 23.

If you want to check integrals you can use Maple (for which Dartmouth has a campus license). For instance, to compute $\int_{-L}^{L} x \sin(n\pi x/L)\, dx$.

```
assume(n,integer);
f := x*sin(n*x*Pi/L);
A := int(f,x=-L..L);
```

Gives answer $2(-1)^{n+1}L^2/n\pi$. How great is that?

p.148-150: #12. Enjoy this beautiful exploration. $r_n(\lambda)$ is the residual (error in the approximation). Try to be rigorous when it says 'show that. . . ', esp. for part c (but don't bother with the full proof by induction for a). For e) please produce a plot of the size of the *relative error* from the 'exact' answer as a function of n the number of expansion terms summed, in the domain 0 to 20. Make your vertical axis a log scale. Fascinating, eh? What n is optimal for the approximation? [Hints: for plotting values vs n in matlab, you should first make a list such as `n=1:20;` then compute everything in terms of this list, *e.g.* `power(10,n)` would be the list $10, 10^2, 10^3, \ldots, 10^{20}$. Note *relative* error means error as a fraction of the answer. The exact answer is given by the `expint` command]

p.214-215: #1 (careful: the $n = 0$ term will need to be treated specially). Isn't it wild that the function $1-x$ has non-zero derivative at the boundary, but the cos's (which have zero derivative there) can approximate it in the mean-square sense?

#3 (explain carefully the missing details of the proof). This result is important later on, and for every mathematician to know.

#5 You will find even and odd separate, so the Gram-Schmidt will be quick. Then only find c_0 and c_1, and write the pointwise error (and do the plot) only for this 2-term approximation. Don't bother computing the max pointwise error or mean-square error.

A: a) Write down *orthonormal* Fourier sine and cosine basis functions on $(-\pi, \pi)$. b) Use the projection formula to compute coefficients $c_n = (f_n, f)$ which give the function $f(x) = x$ on $(-\pi, \pi)$. [Hint: use symmetry to first discard half the coefficients. Also mess around with `http://falstad.com/fourier` for fun.] c) To what value does this Fourier representation converge to at $x = \pi$? d) Apply Parseval's equality to compute $\sum_{n=1}^{\infty} n^{-2}$. Euler first found this value in 1735. [1]

p.219: #2. 'Graph the frequency spectrum' means sketch a stick plot of the first few coefficients c_0, c_1, etc. [see previous hint, and maybe check with `http://integrals.wolfram.com` or Maple.]

p.224-226: #3. If you don't choose to use complex exponentials then you'll need to think explicitly about degeneracy of eigenvalues.

#4. Unfortunately the energy argument won't work so you'll need to try to match BCs for $\lambda < 0$ to show (try to prove) it can or cannot happen. The graphical part is needed since the equation you'll get is transcendental.

[1] See `http://mathworld.wolfram.com/RiemannZetaFunctionZeta2.html`