MATH 46 WORKSHEET : review ODEs in 1 var. 4/2/07 Barnett

Show that the transformation $w = u^{1-n}$ makes the 'Bernoulli eqn'

$$u' + p(t)u = q(t)u^n \quad \leftarrow \text{which looks nonlinear}$$

into a linear eqn. What are the new '$p(t)$' and '$q(t)$'? in the linear eqn?

What method(s) would you use on following? you may need one followed by another.

i) $u'' + 2t(u')^2 = 0$

ii) $u'' + 3u' + 2u = t$

iii) $u'' = 2u + (u')^3$

iv) $u'' + u' = u + \ln t$

v) $\dfrac{u'}{u} = t^2 u^3 + \dfrac{1}{t}$

American Mathematical Society
Committee on Professional Ethics
Procedures Manual

Appendix B: Formal Procedure

The Formal Procedure is followed when COPE decides that it is appropriate or when COPE is directed to do so by the Council. It is rarely used. The following describes this procedure:

1. The Chair of COPE, on the advice of the Committee, and after consultation with the President of the Society, shall appoint a three member Special Committee for the case. At this time any member of COPE with a significant personal interest in the case shall be recused for the duration of the case. Any party to the case may submit to the Chair of COPE a written list of Society members whom they wish not to serve on the Special Committee and reason for the objection. Any party may request certain peer representation on the Special Committee, e.g., a non-tenured member if a party is also non-tenured. The Chair of COPE shall take these suggestions under advisement and may, but is not required to, act upon them. The members of the Special Committee shall generally be knowledgeable of the area of conflict. A time limit for the special investigation will be agreed to by all parties, with the Chair of COPE making the final determination of reasonable time limit in case of non-agreement of the principal parties. Any party may ask for an extension of the time limit from COPE as the case proceeds and the Chair will grant the extension if the request is justified.

2. The Special Committee shall gather facts and statements from all interested parties. Committee members conducting interviews will make written summations of any interviews. After completing what it determines to be a full investigation, and after all parties have been asked to comment on the accuracy and completeness of the record, the Special Committee shall present a written report to COPE. In case parties disagree on statements of facts in the report, their written disagreement shall constitute a part of the report. As part of its report, the Special Committee shall recommend to COPE action that it deems appropriate for the case. All parties to the dispute shall be given copies of the Special Committee report. AMS legal staff shall review the final report of any Special Committee and any final determination and recommendation of any case that goes through a Special Committee procedure.

3. COPE shall review the report of the Special Committee, advice of AMS legal counsel (see note concerning consultation with legal counsel above), and all other relevant information presented to it and shall make a final decision regarding the disposition of the case. It shall take supplemental action as it deems appropriate (cf. Appendix A for examples) and shall communicate its decision and actions to the Council.

SOLUTIONS.

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Show that the transformation $w = u^{1-n}$ makes the 'Bernoulli eqn'

$$u' + p(t)u = q(t)\,u^n$$ ← which looks nonlinear

into a <u>linear eqn</u>. What are the new '$p(t)$' and '$q(t)$' in the linear eqn?

$w = u^{1-n}$ invert to get $u = w^{\frac{1}{1-n}}$ ← (This gave people trouble! Please get back into your algebra)

so $\frac{du}{dt} = \frac{du}{dw}\frac{dw}{dt} = \frac{w^{\frac{n}{1-n}}}{1-n}w'$

b. in ODE: $\frac{w^{\frac{n}{1-n}}}{1-n}w' + p(t)\,w^{\frac{1}{1-n}} = q(t)\left(w^{\frac{1}{1-n}}\right)^n = q(t)\,w^{\frac{n}{1-n}}$

lt. by $\frac{1-n}{w^{\frac{n}{1-n}}}$: $w' + \underbrace{(1-n)p(t)}_{\text{the 'p(t)'}}\,w = \underbrace{(1-n)q(t)}_{\text{the 'q(t)'}}$

What <u>method(s)</u> would you use on following? you may need one followed by another.

i) $u'' + 2t(u')^2 = 0$ $v = u'$ then separate variables (1st order)

ii) $u'' + 3u' + 2u = t$ Und. Coeffs.

iii) $u'' = 2u + (u')^3$ $v = u'$ then $G(u,u',u'')=0$ indep of t, use $G(u, v, v\frac{dv}{du}) = 0$ to get $v(u)$ then get $u(t)$ by 1st-order ODE

iv) $u'' + u' = u + \ln t$ Variation of Parameters.

v) $\frac{u'}{u} = t^2u^3 + \frac{1}{t}$ mult. by u, then it's a Bernoulli Eqn. as above.

More elegantly, implicit differentiate to get $\frac{dw}{dt} = (1-n)u'u^{-n}$ $(*)$

Divide ODE by u^n: $\underbrace{u'u^{-n}}_{\frac{1}{1-n}w',\ \text{by } (*)} + p(t)\underbrace{u^{1-n}}_{w} = q(t)$

same result.