

# MATH 46 WORKSHEET: Dimensional analysis

W 3/28/07

A) Consider the drag force  $F$  on a sphere of radius  $a$ , moving at speed  $v$  in fluid density  $\rho$ .

$M$	}	$F$	$a$	$v$	$\rho$
$L$					
$T$					

i) Fill in the matrix (if don't know dim's of  $F$ , think of Newton's (2<sup>nd</sup> Law), and decide how  $F$  depends on the other parameters.  $F = \dots$

ii) What is  $\pi$ , the dimensionless quantity formed from  $F, a, v, \rho$ ?

iii) What linear algebra operation did you do to get  $\pi$ ?

B) Consider a pulse of energy  $e$  released at the origin at time  $t=0$ . The medium has heat capacity  $c$  (energy per volume per degree), and thermal conductivity  $\kappa$  (power per length per degree). The temperature at distance  $r$  and time  $t$  is  $u$ . (Assume  $u=0$  everywhere before the pulse)

$E$	}	$e$	$r$	$t$	$u$	$c$	$\kappa$	
$L$								
$T$								
$\Theta$								

i) Using fundamental units energy ( $E$ ), length ( $L$ ), time ( $T$ ) and temperature ( $\Theta$ ), fill in the dimensions of the  $m=6$  quantities in the problem.

ii) Find  $p=2$  independent dimensionless quantities. Since there's freedom, choose

$\pi_1$  to not involve  $u$ :  $\pi_1 = \dots$

$\pi_2$  to not involve  $r$ :  $\pi_2 = \dots$

iii) Pi Theorem tells us  $F(\pi_1, \pi_2) = 0$  so  $\pi_2 = g(\pi_1)$

From this get an expression  $u = (\dots)g(\dots)$

iv) If  $r=0$  how does  $u$  scale with  $t$ ?

v) In general how can you get  $p$  from the linear algebra properties of the matrix?