Math 46: Applied Math: Homework 6

due Wed May 9 ... but best if do relevant questions after each lecture

Trying the questions in the strange order I give will be easier.

p.224-226: #4. Unfortunately the energy argument won't work so you'll need to try to match BCs for $\lambda < 0$ to show (try to prove) it can or cannot happen. The graphical part is needed since the equation you'll get is transcendental.

#6. easy.

#7. Amazing what comes out of an innocent little equation.

#8.

p.243-247: #2. This is to show you the linear algebra analogy of what happens for integral equations. However there are typos: c) should read $A\mathbf{x}-5\mathbf{x} = (1, -1/2, 0)^{\mathrm{T}}$ and d) $A\mathbf{x}-5\mathbf{x} = (1, 4, 0)^{\mathrm{T}}$. [Hint: you will find using the unnormalised eigenvectors easier as the $\mathbf{e}_i$ in p.228-229. Quote your coefficients c_i, $i = 1, 2, 3$, which I'll check–you don't even need to write the solution $\mathbf{u}$. c) will be very quick]

Volterra equations:

#9. [Hint: bring out e^t and differentiate to turn into an ODE]

#6. [you'll need Leibniz formula from #1]

#8. Important to make sure you understand what K^n means.

Fredholm equations (mainly):

A. Find the spectrum and eigenfunctions of the integral operator

$$(Ku)(x) = \int_0^1 (1 - 5x^2y^2)u(y)dy.$$

Is $Ku - u = f$ soluble given the function $f(x) = x$? If so, find the solution $u(x)$. Is it unique?

#13. a [Hint: is kernel degenerate? Try integrating to solve. State when a solution exists for the special case of λ],

b (nonlinear but very easy),

c.

#14.

#4. a (you'll have to use addition formula, stare at it for a bit, then it's easy),

b (harder since need graphical solution; don't bother normalizing eigenfunctions),

c (make sure you get cancellation; the Sturm-Liouville you get in the end is well-known),

d.