Math 46: Applied Math: Homework 5

due Wed May 2 . . . but best if do relevant questions after each lecture

Slightly shorter like last week since you had to recover from midterm 1.

p.148-150: #12. Enjoy this beautiful exploration. $r_n(\lambda)$ is the residual (error in the approximation). Try to be a rigorous as possible when it says 'show that. . . ' For e) please produce a plot of the size of the *relative error* from the 'exact' answer as a function of n the number of expansion terms summed, in the domain 0 to 20. Make your vertical axis a log scale. Fascinating, eh? What n is optimal for the approximation? [Hints: for plotting values vs n in matlab, you should first make a list such as `n=1:20;` then compute everything in terms of this list, *e.g.* `power(10,n)` would be the list $10, 10^2, 10^3, \ldots, 10^{20}$. Make sure you understand the concept of *relative error*; ask if not. The exact answer is given by `expint`]

A: Find out if the oscillatory WKB approximation (Eq. above (2.98)) satisfies the differential equation (2.97) *uniformly*, by computing and interpreting the residual $r(x, \varepsilon)$. Can you conclude from this that WKB gives a uniform approximation to the exact solution?

BONUS: can you improve upon this order of residual by using an approximation of the form $y(x) = k(x)^{-1/2} e^{\pm \frac{i}{\varepsilon} \int k(x)dx} [1 + \varepsilon w(x) + \cdots]$? Give $w(x)$ and state what uniform order of residual you now have.

p.214-215: #1 (careful the $n = 0$ term will need to be treated specially). Isn't it wild that the function $1 - x$ has non-zero derivative at the boundary, but the cos's (which have zero derivative there) can approximate it in the mean-square sense?

#3 (explain carefully the missing details of the proof). This result is important later on, and for every mathematician to know.

#5 You will find even and odd separate, so the Gram-Schmidt will be quick. Then only find c_0 and c_1, and write the pointwise error (and do the plot) only for this 2-term approximation. Don't bother computing the max pointwise error or mean-square error.

p.219: #2. 'Graph the frequency spectrum' means sketch a stick plot of the first few coefficients c_0, c_1, etc. [Hint: what is the symmetry? Feel free to use `http://integrals.wolfram.com` or Maple for the $n \geq 1$ case. Also mess around with `http://falstad.com/fourier` for fun.]

p.224-226: #3. If you don't choose to use complex exponentials then you'll need to think explicitly about degeneracy of eigenvalues.

#4. Unfortunately the energy argument won't work so you'll need to try to match BCs for $\lambda < 0$ to show (try to prove) it can or cannot happen. The graphical part is needed since the equation you'll get is transcendental.