# Math 46: Applied Math: Homework 2

due Wed Apr 11 ... but best if do relevant questions after each lecture

Always check how many terms the question asks for, *e.g.* $y_0 + \varepsilon y_1$ is 2-term.

**p.40-44**: #5. A warm-up question (no pun intended). In b please group together the exponentials in the term involving an integral; this convolution result is called *Duhamel's principle*.

**p.52-54**: #6. You will see in c why this is called a 'pitchfork bifurcation'.

**p.62**: #1.

**p.67-68**: #2. Try to visualize how the two eigenvalues move in the complex plane as $b$ varies. Note you don't need a full solution for each case of $b$, just discussion of behaviour (type of critical point), including the equal-roots case.

#6. Nice connection to 1-variable ODEs here.

**p.79-82**: #1 a.

#12. a, c. For c use `pplane` applet, and enjoy launching many trajectories. Attach a print-out to your homework.

**p.100-104**: #1. This is a quick and easy review of Lecture 2.

#2. This is a lovely example. Please leave enough time to get it right and produce the plots—you will love it when it works. First ask yourself, is the unperturbed ODE oscillatory or decaying/growing? You will find the ICs given cause the unperturbed solution to be special (how?), and the perturbation messes this up in a dramatic way. Please don't bother finding, or plotting, the Taylor series. Instead produce the following two plots at $\varepsilon = 0.04$:

- compare $u(t)$, $u_0(t)$, $\varepsilon u_1(t)$, and $u_a(t)$ on the same axes in the domain $t \in [0, 5]$
- show error $E(\varepsilon, t) := u_a(t) - u(t)$ in the domain $t \in [0, 3]$, making sure your axes illustrate its size

You should find the error is very small, staying much smaller than $10^{-3}$ in most of the latter domain. If you don't find this, you'll need to debug your algebra! [*e.g.* make sure $u_1(t)$ satisfies the correct ICs]

#4 (easy algebra review; remember to substitute for $y$!)

#11. (connects to the planet-projectile ODE scaling problem from Lecture 3). Getting the 3rd term involves some high powers of $t$; do not be alarmed. However, only compute $t_m$ and $h_{max}$ to order $\varepsilon$ since order $\varepsilon^2$ is an algebra nightmare.