A Little Behind

- Recall that last week's homework together with today's assignment is due Wednesday.
- We meet tomorrow in our x-hour from 12:15–1:05.
- Starting tomorrow, this week's assignments are due Monday.

It has a name!

Theorem (Riemann's Theorem)

Suppose that g is continuous on a contour Γ. Let $D = \{ z \in \mathbf{C} : z \notin \Gamma \}$. For each $n = 1, 2, 3, \dots,$ define

$$F_n(z) = \int_\Gamma \frac{g(\omega)}{(\omega - z)^n}\, d\omega \quad \text{for } z \in D.$$

Then F_n is analytic on D and for each n,

$$F_n'(z) = nF_{n+1}(z) = n \int_\Gamma \frac{g(\omega)}{(\omega - z)^{n+1}}\, d\omega.$$

Corollary (The Big Payoff)

If f is analytic in a domain D, then f' is analytic in D. Hence f has derivatives of all orders throughout D.

We actually proved something stronger.

Theorem (Cauchy's Integral Formula for the Derivatives)

Suppose that f is analytic on and inside a simple closed contour Γ. Let D be the interior of Γ. Then for all $n \geq 0$, $f^{(n)}(z)$ exists for all $z \in D$ and

$$f^{(n)}(z) = \frac{n!}{2\pi i} \int_\Gamma \frac{f(\omega)}{(\omega - z)^{n+1}}\, d\omega. \quad \textit{for all } z \in D.$$

Remark

Note that the case $n = 0$ is just the usual Cauchy Integral Formula.

Corollary

If $f(z) = u(z) + iv(z)$ is analytic in a domain D. Then u and v have continuous partials of all orders in D. In particular, u and v are always harmonic.

Corollary

If u is harmonic in a domain D, then u has continuous paritials of all orders in D.

Corollary (HW)

If u is harmonic on a simply connected domain D, then u has a harmonic conjugate on D.