Last Time

Theorem

Suppose that $p(z) \in \mathbb{C}[z]$ has real coefficients and that $p(z) \geq 1$. Then there are real numbers $r_1, \ldots, r_s$ and irreducible quadradics $q_1(z), \ldots, q_k(z)$ such that $p(z) = r + 2k$ and

$$p(z) = a(z - r_1) \cdots (z - r_s) q_1(z) \cdots q_k(z).$$

Theorem

Suppose that u is Harmonic in a Domain D.

1. *If v and w are Harmonic conjugates of u in D, then v and w differ by a real constant.*
2. *There may not be a Harmonic conjugate for u in all of D.*
3. *If D is an open disk, then u has a harmonic conjugate in D.*

Partial Fraction Decompositions

Theorem (§3.1, Theorem 2)

Suppose that

$$R(z) = \frac{p(z)}{q(z)} = \frac{p(z)}{a(z-w_1)^{d_1}(z-w_2)^{d_2}\cdots(z-w_s)^{d_s}}$$

is a rational function with $\{ w_1, \ldots, w_s \}$ *distinct and* $p(z) < q(z) = d_1 + \cdots d_s$. *Then*

$$R(z) = r_1(z) + \cdots + r_s(z)$$

with

$$r_k(z) = \frac{A_0^{(k)}}{(z-w_k)^{d_k}} + \frac{A_1^{(k)}}{(z-w_k)^{d_k-1}} + \cdots + \frac{A_{d_k-1}^{(k)}}{z-w_k}$$

for complex constants $A_j^{(k)}$.

Example

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$$\frac{4z-4}{z(z-1)(z-2)^2} = -\frac{1}{z} + \frac{8}{z-1} + \frac{6}{(z-2)^2} - \frac{7}{z-2}.$$

Theorem (§3.1, Equation (21))

In general, if $R(z)$ has the form in the Decomposition Theorem, then

$$A_j^{(k)} = \lim_{z \to w_k} \frac{1}{j!} \frac{d^j}{dz^j} \left((z-w_k)^{d_j} R(z) \right).$$