Last Time

Definition

If γ is a smooth curve with admissible parameterization $z : [a, b] \to \mathbb{C}$ given by $z(t) = x(t) + iy(t)$, then the length of γ is

$$\ell(\gamma) := \int_a^b |z'(t)|\, dt = \int_a^b \sqrt{x'(t)^2 + y'(t)^2}\, dt$$

If $\Gamma = \gamma_1 + \cdots + \gamma_n$ is a contour, then

$$\ell(\Gamma) = \ell(\gamma_1) + \cdots + \ell(\gamma_n).$$

Remark

We know from multivariable calculus that $\ell(\gamma)$, and hence $\ell(\Gamma)$, is independent of admissible parameterization.

Ordinary Inegrals

Definition

If $z(t) = u(t) + iv(t)$ and $z : [a, b] \to \mathbb{C}$ is continuous, then we define

$$\int_a^b z(t)\,dt = \int_a^b u(t)\,dt + i \int_a^b v(t)\,dt.$$

Since these are just ordinary integrals, if $F'(t) = z(t)$, then

$$\int_a^b z(t)\,dt = F(t)\,\Big|_a^b = F(b) - F(a).$$

Example

If $z(t) = e^{at}$, then $z'(t) = ae^{at}$ for any $a \in \mathbb{C}$. Hence

$$\int_0^{\frac{\pi}{2}} e^{2it}\,dt = \frac{e^{2it}}{2i}\,\Big|_0^{\frac{\pi}{2}} = i.$$