Worksheet – Functions – Math 3 – Jan 4, 2012

One way to build functions is by composition, i.e. plugging one function into another. If $f(x)$ and $g(x)$ are functions, then for whatever x for which $g(x)$ is in the domain of $f(x)$, then we can write

$$(f \circ g)(x) = f(g(x)).$$

For example, if $f(x) = \frac{x+1}{3x-2}$ and $g(x) = \sqrt{x}$, then

$$(f \circ g)(x) = \frac{\sqrt{x}+1}{3\sqrt{x}-2} \quad (\text{for } x \neq \pm(2/3)^2)$$

and

$$(g \circ f)(x) = \sqrt{\frac{x+1}{3x-2}} \quad (\text{whenever } \frac{x+1}{3x-2} \geq 0).$$

1. Let $f(x) = \dfrac{x+1}{3x-2}$ and $g(x) = \dfrac{1}{x}$.

   (a) Calculate $(f \circ g)(x)$ and $(g \circ f)(x)$.

   (b) What is the domain of $(g \circ f)(x)$?
   [hint: Careful! The domain of $(g \circ f)(x)$ is the set of x's which satisfy *both* (1) $f(x)$ exists, and (2) $(g \circ f)(x)$ exists.]

2. Let $f = \frac{x+1}{3x-2}$

   (a) Calculate $f^{-1}(x)$.

   (b) Check your answer to #1 by explicitly calculating $(f \circ f^{-1})(x)$ and $(f \circ f^{-1})(x)$ (you should get x both times).

(c) If $(f \circ g)(x) = x + 2$, what is $g(x)$?
[hint: since $(f \circ g)(x) = f(g(x)) = x + 2$, we know

$$g(x) = f^{-1}(f(g(x)) = f^{-1}(x + 2).]$$

Answers:

1. (a) $(f \circ g)(x) = \frac{\frac{1}{x}+1}{3\frac{1}{x}-2}$, $(g \circ f)(x) = \frac{3x-2}{x+1}$
   (b) All $x \neq 2/3, -1$, i.e. $(-\infty, -1) \cup (-1, 2/3) \cup (2/3, \infty)$
2. (a) $f^{-1}(x) = \frac{2x+1}{3x-1}$
   (b) (calculation)
   (c) $g(x) = \frac{2x+5}{3x+5}$