Derivatives of inverse functions

More on implicit differentiation

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Every time:

(1) Take $\frac{d}{dx}$ of both sides.

(2) Add and subtract to get the $\frac{dy}{dx}$ on one side and everything else on the other.

(3) Factor out $\frac{dy}{dx}$ and divide both sides by its coefficient.

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Example Suppose $\cos(y) = x + y$.

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Notice:

$$\frac{dy}{dx} = 1/\left(\frac{dx}{dy}\right)$$

This is true in general!

Using implicit differentiation for good: Inverse functions.

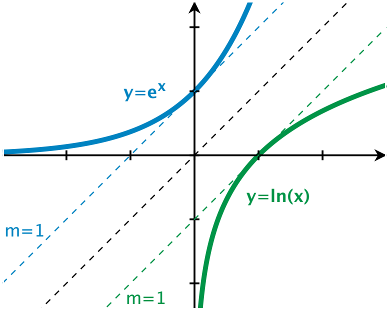
The Derivative of $y = \ln x$

Remember:

(1) $y = e^x$ has a slope through the point (0,1) of 1.

(2) The natural log is the *inverse* to e^x , so

$$y = \ln x \quad \Longrightarrow \quad e^y = x$$



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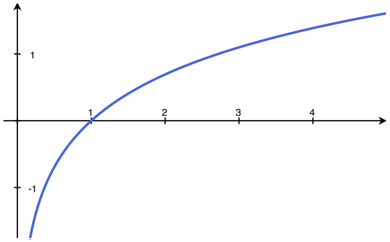
$$\boxed{\frac{d}{dx} \ln(x) = \frac{1}{x}}$$

Does it make sense?

$$\frac{d}{dx}\ln(x) = \frac{1}{x}$$

$f(x) = \ln(x)$



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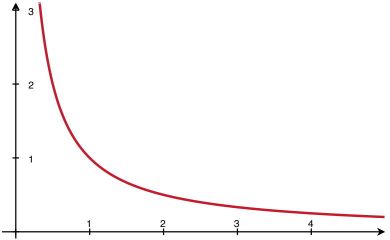

Examples

Calculate

1. $\frac{d}{dx} \ln x^2$

2. $\frac{d}{dx} \ln(\sin(x^2))$

3. $\frac{d}{dx} \log_3(x)$

   [hint: $\log_a x = \frac{\ln x}{\ln a}$]

Examples

Calculate

1. $\frac{d}{dx} \ln x^2 = \frac{2x}{x^2} = \frac{2}{x}$

2. $\frac{d}{dx} \ln(\sin(x^2)) = \frac{2x\cos(x^2)}{\sin(x^2)}$

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Notice, every time:

$$\frac{d}{dx} \ln(f(x)) = \frac{f'(x)}{f(x)}$$

Back to inverses

In the case where $y = \ln(x)$, we used the fact that $\ln(x) = f^{-1}(x)$, where $f(x) = e^x$, and got

$$\frac{d}{dx}\ln(x) = \frac{1}{e^{\ln(x)}}.$$

In general, calculating $\frac{d}{dx}f^{-1}(x)$:

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(1) Rewrite $y = f^{-1}(x)$ as $f(y) = x$.

(2) Use implicit differentiation:

$$f'(y) * \frac{dy}{dx} = 1 \qquad \text{so} \qquad \boxed{\frac{dy}{dx} = \frac{1}{f'(y)} = \frac{1}{f'(f^{-1}(x))}}.$$

Examples

Just to check, use the rule

$$\frac{d}{dx} f^{-1}(x) = \frac{1}{f'(f^{-1}(x))}$$

to calculate

1. $\frac{d}{dx} \ln(x)$ (the inverse of e^x)

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   $$\boxed{\frac{d}{dx}\sqrt{x} = \frac{1}{2 * (\sqrt{x})}} \quad ☺$$

Inverse trig functions

Two notations:

$f(x)$	$f^{-1}(x)$
$\sin(x)$	$\sin^{-1}(x) = \arcsin(x)$
$\cos(x)$	$\cos^{-1}(x) = \arccos(x)$
$\tan(x)$	$\tan^{-1}(x) = \arctan(x)$
$\sec(x)$	$\sec^{-1}(x) = \mathrm{arcsec}(x)$
$\csc(x)$	$\csc^{-1}(x) = \mathrm{arccsc}(x)$
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There are lots of points we know on these functions...

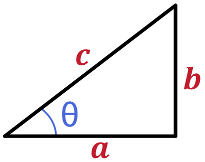
Examples:

1. Since $\sin(\pi/2) = 1$, we have $\arcsin(1) = \pi/2$
2. Since $\cos(\pi/2) = 0$, we have $\arccos(0) = \pi/2$

Etc...

In general:

arc___(-) takes in a ratio and spits out an angle:



$\cos(\theta) = a/c$ so $\arccos(a/c) = \theta$

$\sin(\theta) = b/c$ so $\arcsin(b/c) = \theta$

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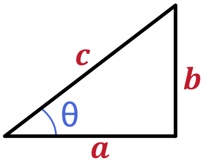

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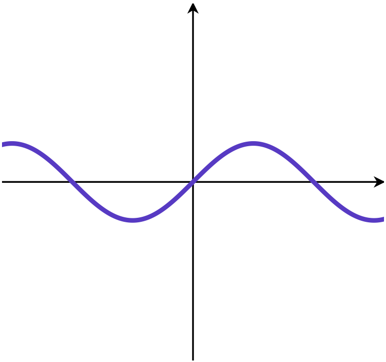
Domain problems:

$\sin(0) = 0, \quad \sin(\pi) = 0, \quad \sin(2\pi) = 0, \quad \sin(3\pi) = 0, \ldots$

So which is the right answer to arcsin(0), really?

Domain/range

$$y = \sin(x)$$



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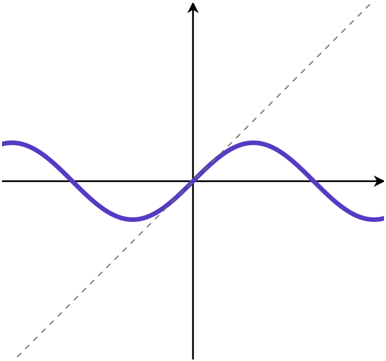

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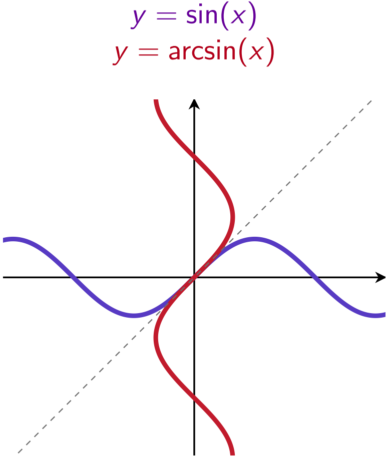

Domain: $-1 \leq x \leq 1$

Domain/range

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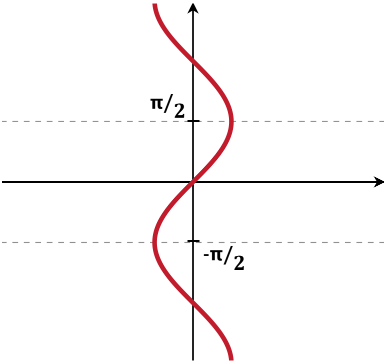

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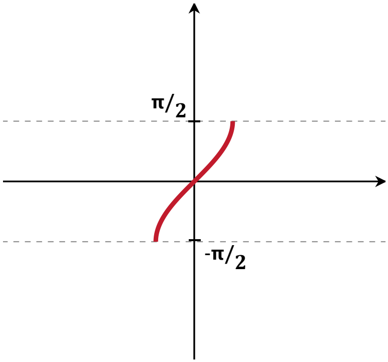
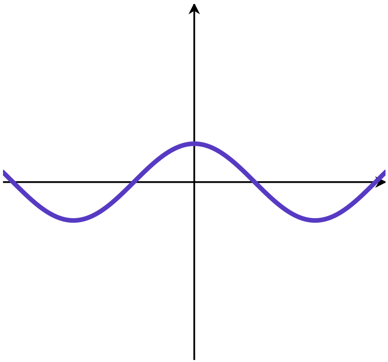


Domain: $-1 \leq x \leq 1$ Range: $-\pi/2 \leq y \leq \pi/2$

Domain/range

$$y = \cos(x)$$



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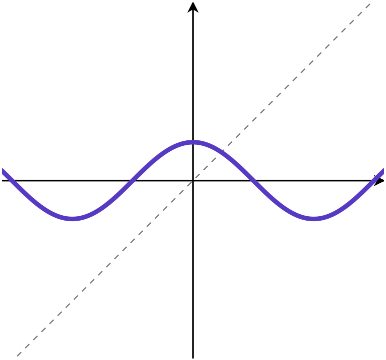

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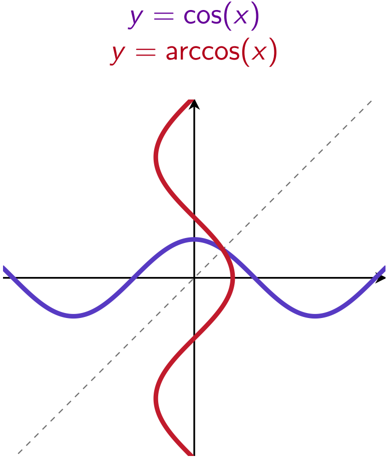

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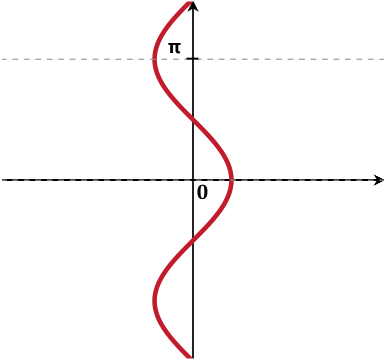

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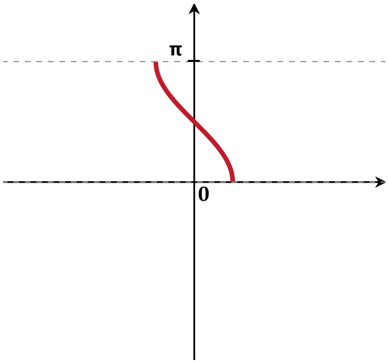
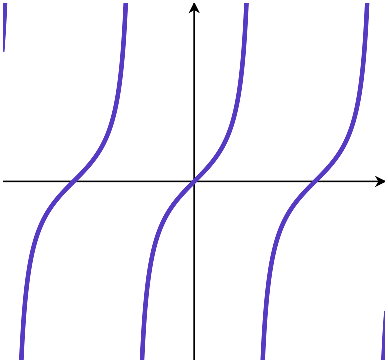


Domain: $-1 \leq x \leq 1$ Range: $0 \leq y \leq \pi$

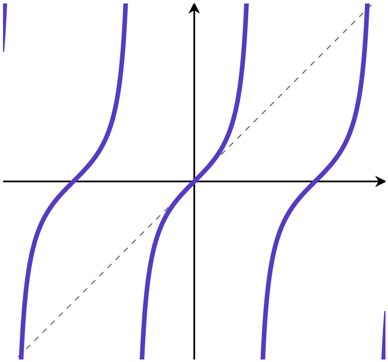
Domain/range

$y = \tan(x)$



Domain/range

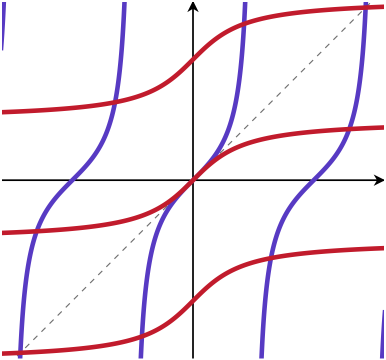
$$y = \tan(x)$$



Domain/range

$y = \tan(x)$

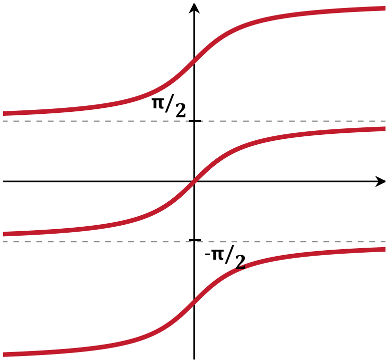
$y = \arctan(x)$



Domain: $-\infty \leq x \leq \infty$

Domain/range

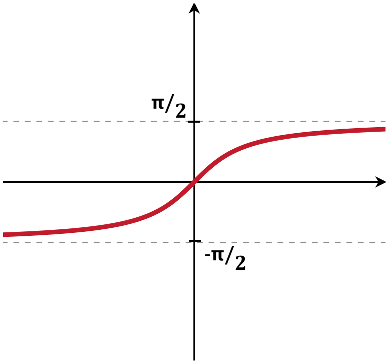
$y = \arctan(x)$



Domain: $-\infty \leq x \leq \infty$

Domain/range

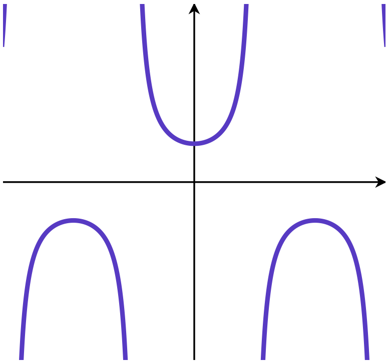
$y = \arctan(x)$



Domain: $-\infty \leq x \leq \infty$ Range: $-\pi/2 < y < \pi/2$

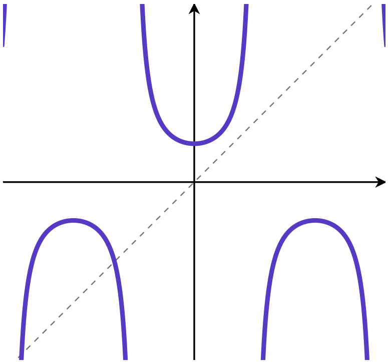
Domain/range

$y = \sec(x)$



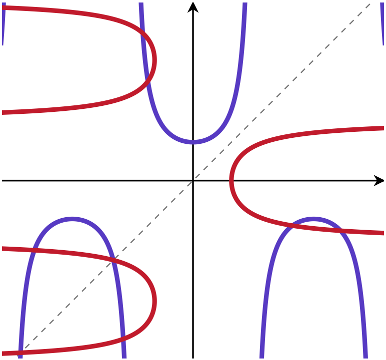
Domain/range

$y = \sec(x)$



Domain/range

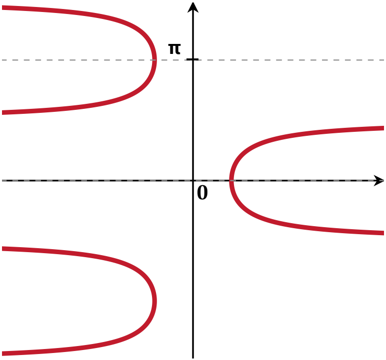
$y = \sec(x)$
$y = \operatorname{arcsec}(x)$



Domain: $x \leq -1$ and $1 \leq x$

Domain/range

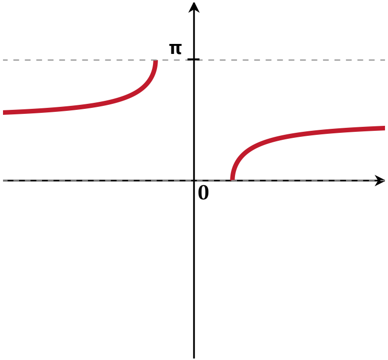
$y = \text{arcsec}(x)$



Domain: $x \leq -1$ and $1 \leq x$

Domain/range

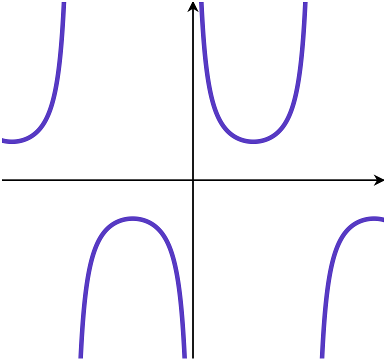
$y = \text{arcsec}(x)$



Domain: $x \leq -1$ and $1 \leq x$ Range: $0 \leq y \leq \pi$

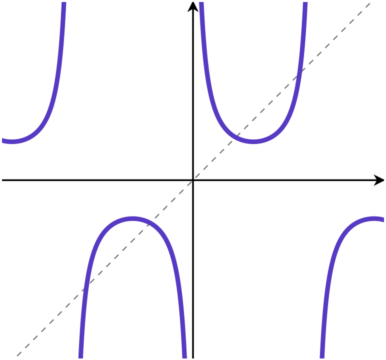
Domain/range

$$y = \csc(x)$$

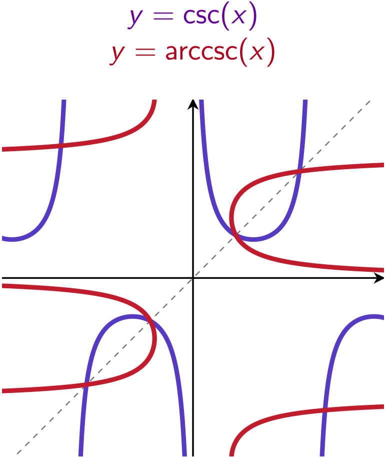


Domain/range

$$y = \csc(x)$$



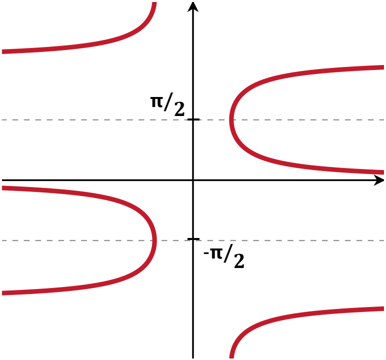
Domain/range



Domain: $x \leq -1$ and $1 \leq x$

Domain/range

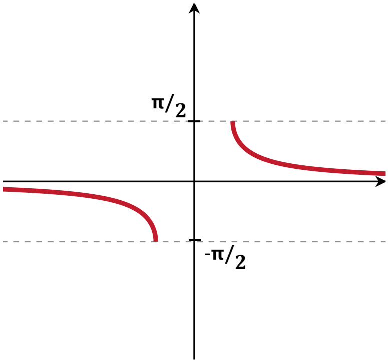
$y = \text{arccsc}(x)$



Domain: $x \leq -1$ and $1 \leq x$

Domain/range

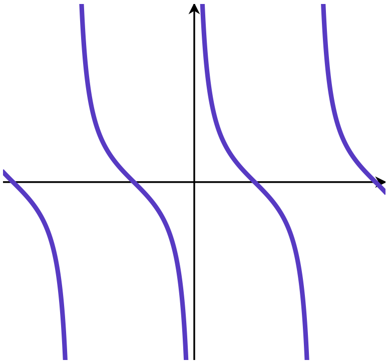
$y = \text{arccsc}(x)$



Domain: $x \leq -1$ and $1 \leq x$ Range: $-\pi/2 \leq y \leq \pi/2$

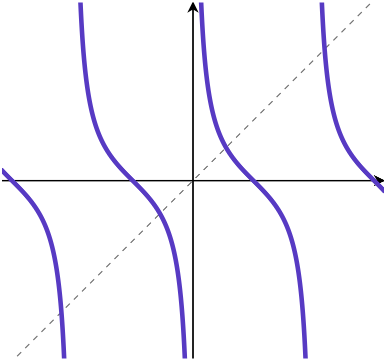
Domain/range

$$y = \cot(x)$$



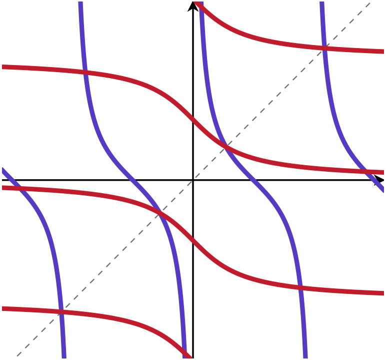
Domain/range

$$y = \cot(x)$$



Domain/range

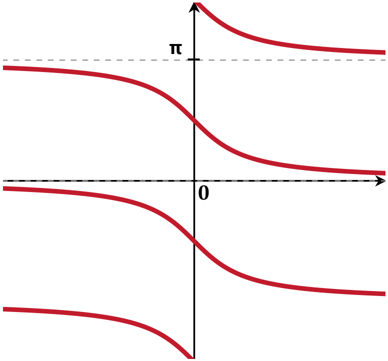
$y = \cot(x)$
$y = \text{arccot}(x)$



Domain: $-\infty \leq x \leq \infty$

Domain/range

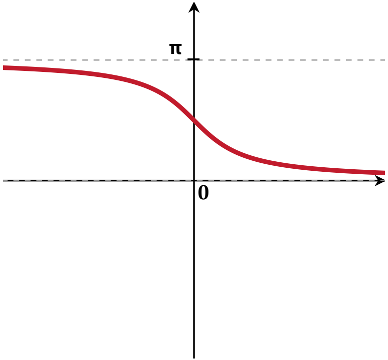
$y = \text{arccot}(x)$



Domain: $-\infty \leq x \leq \infty$

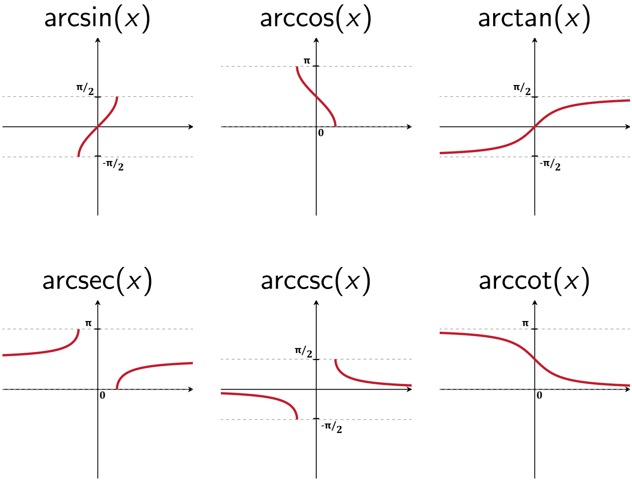
Domain/range

$y = \text{arccot}(x)$



Domain: $-\infty \leq x \leq \infty$ Range: $0 < y < \pi$

Graphs



arcsin(x)
π/2
-π/2
arccos(x)
π
0
arctan(x)
π/2
-π/2
arcsec(x)
π
0
arccsc(x)
π/2
-π/2
arccot(x)
π
0

Recall:

$f(x)$	$f'(x)$
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$
$\tan(x)$	$\sec^2(x)$
$\sec(x)$	$\sec(x)\tan(x)$
$\csc(x)$	$-\csc(x)\cot(x)$
$\cot(x)$	$-\csc^2(x)$

Back to Derivatives

Use implicit differentiation to calculate the derivatives of

1. $\arcsin(x)$
2. $\arctan(x)$

Use the rule

$$\frac{d}{dx} f^{-1}(x) = \frac{1}{f'(f^{-1}(x))}$$

to check your answers, and then to calculate the derivatives of the other inverse trig functions:

1. $\frac{d}{dx} \arccos(x)$
2. $\frac{d}{dx} \text{arcsec}(x)$
3. $\frac{d}{dx} \text{arccsc}(x)$
4. $\frac{d}{dx} \text{arccot}(x)$

Back to Derivatives

Use implicit differentiation to calculate the derivatives of

1. $\arcsin(x) = \frac{1}{\cos(\arcsin(x))}$
2. $\arctan(x) = \frac{1}{\sec^2(\arctan(x))}$

Use the rule

$$\frac{d}{dx} f^{-1}(x) = \frac{1}{f'(f^{-1}(x))}$$

to check your answers, and then to calculate the derivatives of the other inverse trig functions:

1. $\frac{d}{dx} \arccos(x) = -\frac{1}{\sin(\arccos(x))}$
2. $\frac{d}{dx} \text{arcsec}(x) = \frac{1}{\sec(\text{arcsec}(x)) \tan(\text{arcsec}(x))}$
3. $\frac{d}{dx} \text{arccsc}(x) = -\frac{1}{\csc(\text{arccsc}(x)) \cot(\text{arccsc}(x))}$
4. $\frac{d}{dx} \text{arccot}(x) = -\frac{1}{\csc^2(\text{arccot}(x))}$

Using implicit differentiation to calculate $\frac{d}{dx}\arcsin(x)$

$$\boxed{\text{If } y = \arcsin(x) \text{ then } x = \sin(y).}$$

Take $\frac{d}{dx}$ of both sides of $x = \sin(y)$:

$$\text{Left hand side:} \qquad \frac{d}{dx}x = 1$$

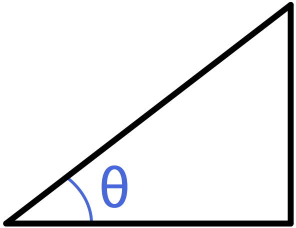
Right hand side:
$$\frac{d}{dx}\sin(y) = \cos(y) * \frac{dy}{dx} = \cos(\arcsin(x)) * \frac{dy}{dx}$$

So

$$\boxed{\frac{dy}{dx} = \frac{1}{\cos(\arcsin(x))}.}$$

Simplifying $\cos(\arcsin(x))$

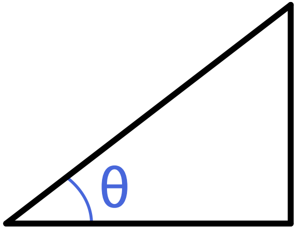
Call $\arcsin(x) = \theta$.



Simplifying $\cos(\arcsin(x))$

Call $\arcsin(x) = \theta$.

$\sin(\theta) = x$



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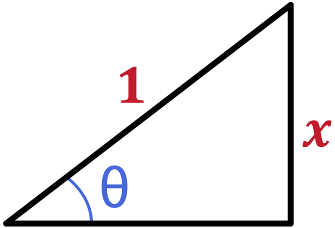
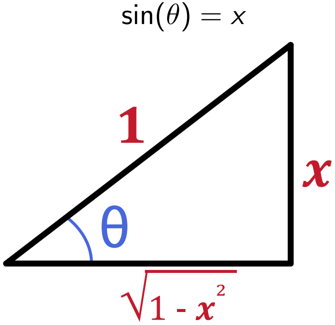


Key: This is a simple triangle to write down whose angle θ has $\sin(\theta) = x$

Simplifying cos(arcsin(x))

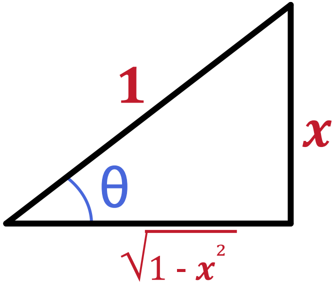
Call $\arcsin(x) = \theta$.



Simplifying cos(arcsin(x))

Call arcsin(x) $= \theta$.

$\sin(\theta) = x$

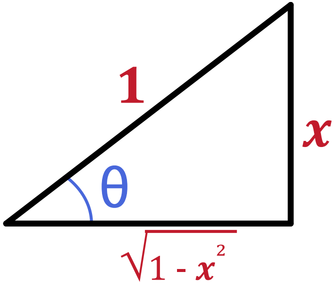


So $\cos(\theta) = \sqrt{1 - x^2}/1$

Simplifying $\cos(\arcsin(x))$

Call $\arcsin(x) = \theta$.

$\sin(\theta) = x$

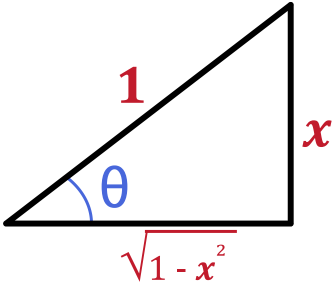


So $\cos(\arcsin(x)) = \sqrt{1 - x^2}$

Simplifying $\cos(\arcsin(x))$

Call $\arcsin(x) = \theta$.

$\sin(\theta) = x$



So $\quad \cos(\arcsin(x)) = \sqrt{1 - x^2}$

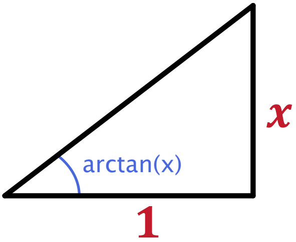
So $\quad \dfrac{d}{dx} \arcsin(x) = \dfrac{1}{\cos(\arcsin(x))} = \dfrac{1}{\sqrt{1 - x^2}}.$

Calculating $\frac{d}{dx}\arctan(x)$.

We found that

$$\frac{d}{dx}\arctan(x) = \frac{1}{\sec^2(x)} = \left(\frac{1}{\sec(x)}\right)^2$$

Simplify this expression using



Calculating $\frac{d}{dx}\arctan(x)$.

We found that

$$\frac{d}{dx}\arctan(x) = \frac{1}{\sec^2(x)} = \left(\frac{1}{\sec(x)}\right)^2$$

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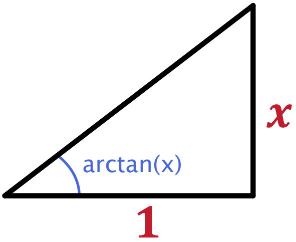

$$\frac{dy}{dx} = \left(\frac{1}{\sec(\arctan(x))}\right)^2 = \frac{1}{1+x^2}$$

To simplify the rest, use the triangles