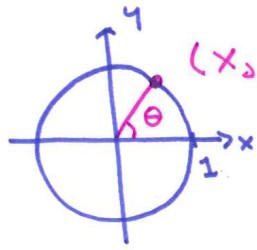


Trig Identities :



$$(x, y) = (\cos \theta, \sin \theta)$$

Resulting identities:

$$\cos(-\theta) = \cos \theta$$

$$\sin(-\theta) = -\sin \theta$$

$$\cos^2(\theta) + \sin^2 \theta = 1$$

Other

useful identities: _____

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

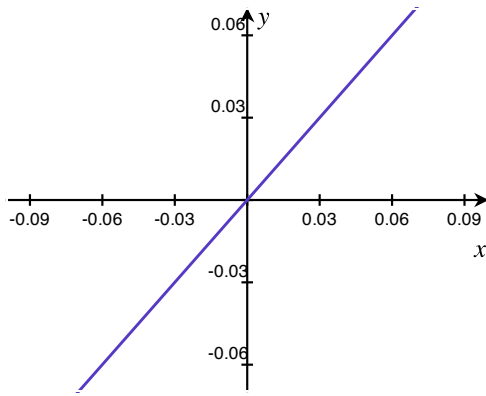
$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \sin(\beta)\cos(\alpha)$$

The derivative of sine

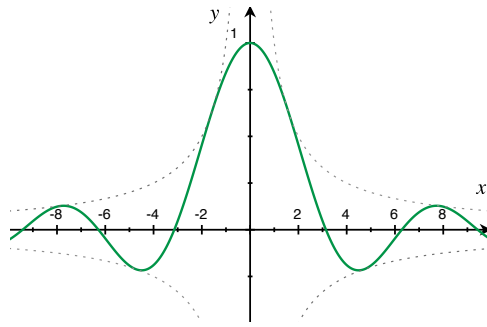
$$\begin{aligned} \frac{d}{dx} \sin x &= \lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sin(x)\cos(h) + \cos(x)\sin(h) - \sin(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sin(x)(\cos(h) - 1) + \cos(x)\sin(h)}{h} \\ &= \sin(x) \lim_{h \rightarrow 0} \frac{\cos(h) - 1}{h} + \cos(x) \lim_{h \rightarrow 0} \frac{\sin(h)}{h} \end{aligned}$$

Recall: $\cos(0) = 1$ and $\sin(0) = 0$

Near $x = 0$, $\sin(x) \approx x$:

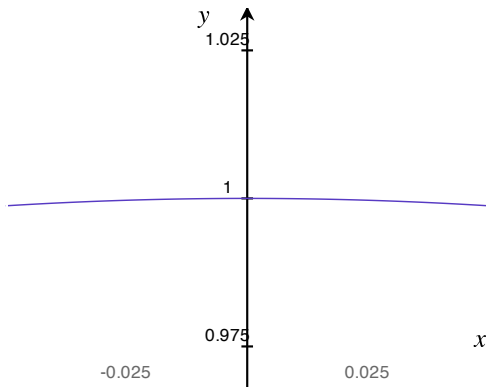


Graph of $\frac{\sin(x)}{x}$:

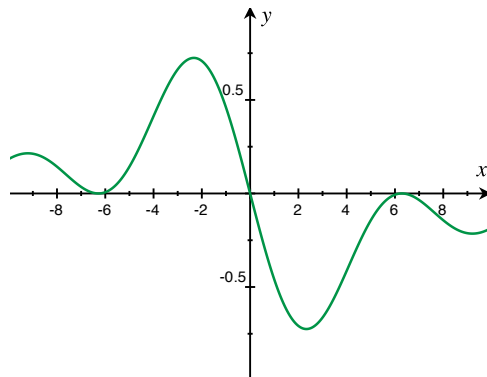


$$\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = 1$$

Near $x = 0$, $\cos(x) \approx 1$:



Graph of $\frac{\cos(x)-1}{x}$:



$$\lim_{x \rightarrow 0} \frac{\cos(x) - 1}{x} = 0$$

The derivative of sine

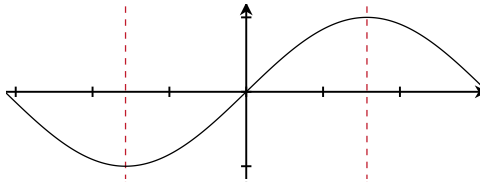
$$\begin{aligned}\frac{d}{dx} \sin x &= \lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sin(x)\cos(h) + \cos(x)\sin(h) - \sin(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sin(x)(\cos(h) - 1) + \cos(x)\sin(h)}{h} \\ &= \sin(x) \lim_{h \rightarrow 0} \frac{\cos(h) - 1}{h} + \cos(x) \lim_{h \rightarrow 0} \frac{\sin(h)}{h} \\ &= \sin(x) * 0 + \cos(x) * 1 \\ &= \boxed{\cos(x)}\end{aligned}$$

The derivative of cosine

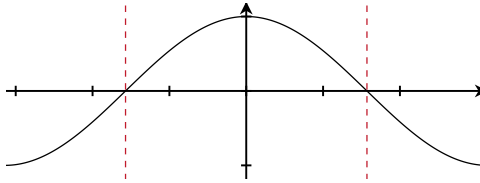
$$\begin{aligned}\frac{d}{dx} \cos x &= \lim_{h \rightarrow 0} \frac{\cos(x+h) - \cos(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\cos(x)\cos(h) - \sin(x)\sin(h) - \cos(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\cos(x)(\cos(h) - 1) - \sin(x)\sin(h)}{h} \\ &= \cos(x) \lim_{h \rightarrow 0} \frac{\cos(h) - 1}{h} - \sin(x) \lim_{h \rightarrow 0} \frac{\sin(h)}{h} \\ &= \cos(x) * 0 - \sin(x) * 1 \\ &= \boxed{-\sin(x)}\end{aligned}$$

Does it make sense?

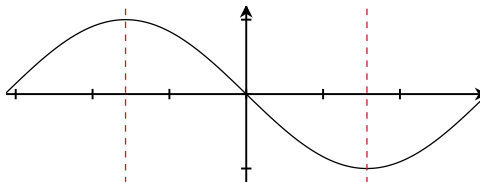
$$y = \sin(x) :$$



$$y = \cos(x) :$$



$$y = -\sin(x) :$$



Examples

Calculate...

1. $\frac{d}{dx} \sin(2x)$

2. $\frac{d}{dx} \cos(3x + \sqrt{x})$

3. $\frac{d}{dx} \sin(x) \cos(x)$

Notice: $\sin(x) \cos(x) = \frac{1}{2} \sin(2x)$, and $\cos^2(x) - \sin^2(x) = \cos(2x)$.

Does your answer still make sense from this perspective?

4. $\frac{d}{dx} \sin(\cos(x^2 + 2))$

On your own, fill in the rest of the trig functions:

$$1. \frac{d}{dx} \tan(x) = \frac{d}{dx} \frac{\sin(x)}{\cos(x)}$$

$$2. \frac{d}{dx} \cot(x) = \frac{d}{dx} \frac{\cos(x)}{\sin(x)}$$

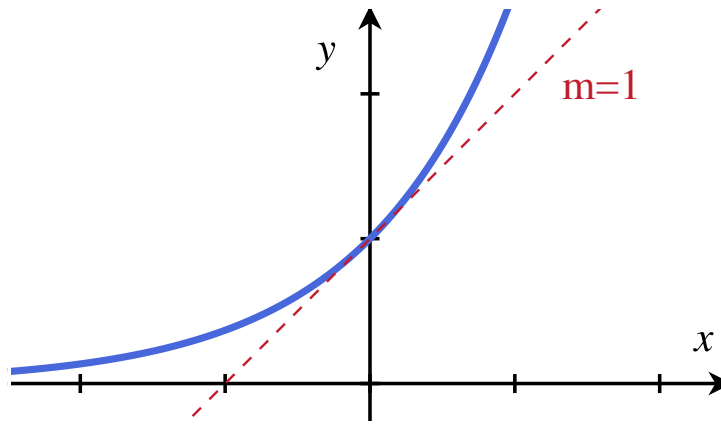
$$3. \frac{d}{dx} \sec(x) = \frac{d}{dx} (\cos(x))^{-1}$$

$$4. \frac{d}{dx} \csc(x) = \frac{d}{dx} (\sin(x))^{-1}$$

The Derivative of $y = e^x$

Recall!

e^x is the unique exponential function whose slope at $x = 0$ is 1:



$$\lim_{h \rightarrow 0} \frac{e^{0+h} - e^0}{h} = \lim_{h \rightarrow 0} \frac{e^h - 1}{h} = 1$$

The Derivative of $y = e^x$...

$$\lim_{h \rightarrow 0} \frac{e^h - 1}{h} = 1$$

$$\begin{aligned} \frac{d}{dx} e^x &= \lim_{h \rightarrow 0} \frac{e^{x+h} - e^x}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^x (e^h - 1)}{h} \\ &= e^x \lim_{h \rightarrow 0} \frac{e^h - 1}{h} \\ &= e^x * 1 \end{aligned}$$

So

$$\frac{d}{dx} e^x = e^x$$

Examples

Calculate...

1. $\frac{d}{dx} e^{17x}$

2. $\frac{d}{dx} e^{x \ln(3)}$

3. $\frac{d}{dx} e^{\sin x}$

4. $\frac{d}{dx} e^{\sqrt{x^2+x}}$

Other bases

We know $\frac{d}{dx}x^a = ax^{a-1}$, but what is $\frac{d}{dx}a^x$?

Notice:

$$a^x = e^{\ln(a^x)} = e^{x \ln(a)}$$

So

$$\frac{d}{dx}a^x = \frac{d}{dx}e^{x \ln(a)} = \ln(a)e^{x \ln(a)} = \ln(a) * a^x$$

Example: $\frac{d}{dx}2^x = \ln(2) * 2^x$

Sanity check: $\frac{d}{dx}e^x \stackrel{?}{=} \ln(e) * e^x = 1 * e^x \quad \text{☺}$

Derivative so far

Definition: $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

Combining functions:

$$\frac{d}{dx}c * f(x) = c * f'(x) \quad \frac{d}{dx}(f(x) + g(x)) = f'(x) + g'(x)$$

$$\frac{d}{dx}(f(x) * g(x)) = f(x)g'(x) + g(x)f'(x) \quad \frac{d}{dx}(f(g(x))) = f'(g(x)) * g'(x)$$

Basic functions:

$f(x)$	x^a	$\sin(x)$	$\cos(x)$	e^x
$f'(x)$	ax^{a-1}	$\cos(x)$	$-\sin(x)$	e^x

Other functions:

$f(x)$	$\tan(x)$	$\cot(x)$	$\sec(x)$	$\csc(x)$	a^x
$f'(x)$	$\sec^2(x)$	$-\csc^2(x)$	$\sec(x)\tan(x)$	$-\csc(x)\cot(x)$	$\ln(a)a^x$

Example

Compute the derivatives of

1. $e^{17x^2+\sqrt{x}}$

2. $\tan\left(e^{17x^2+\sqrt{x}}\right)$

3. $\csc(x) * \left[3^x + \tan^3\left(e^{17x^2+\sqrt{x}}\right)\right]^4$