## Guidelines for Sketching a Curve

| Properties to Look for | EXPLANATION | $f(x)=\frac{2 x^{2}}{x^{2}-1}$ |
| :---: | :---: | :---: |
| 1. Domain | All $x$ where $f(x)$ is defined | $x \neq \pm 1$ |
| 2. $x$ - and $y$-intercepts | $x$-intercepts: $f(x)=0$ <br> $y$-intercepts: $f(0)$ | $\begin{aligned} & x=0 \\ & y=0 \end{aligned}$ |
| 3. Symmetries | even: $f(-x)=f(x)$ <br> odd: $f(-x)=-f(x)$ <br> periodic: $f(x+p)=f(x)$ | even |
| 4. Asymptotes | $\begin{aligned} & \text { horizontal: } \quad y=\lim _{x \rightarrow \pm \infty} f(x) \\ & \text { vertical: } \quad x=a \underline{\text { if }} \lim _{x \rightarrow a^{ \pm}} f(x)= \pm \infty \end{aligned}$ | $\begin{gathered} y=2 \\ x=-1 \text { and } x=1 \end{gathered}$ |
| 5. Increases or Decreases (I/D-Test) | increases: decreases: $\begin{aligned} & f^{\prime}(x)>0 \\ & f^{\prime}(x)<0 \end{aligned}$ | $\begin{gathered} (-\infty,-1) \text { and }(-1,0) \\ (0,1) \text { and }(1, \infty) \end{gathered}$ |
| 6. Local Maxima and Minima (1st or 2nd Derivative Test) | maximum: $f^{\prime}$ from + to - at $x=c$ <br> minimum: $f^{\prime}$ from - to + at $x=c$ | $x=0$ <br> none |
| 7. Concavity and Inflections (Concavity Test) | concave upward: $\quad f^{\prime \prime}(x)>0$ concave downward: $\quad f^{\prime \prime}(x)<0$ inflection point: $\quad f^{\prime \prime}$ changes sign | $\begin{gathered} (-\infty,-1) \text { and }(1, \infty) \\ (-1,1) \\ \text { none } \end{gathered}$ |
| 8. Sketch the Curve |  |  |

