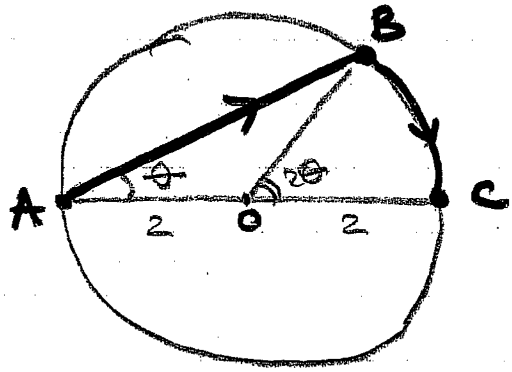
Section 4.7

Problem 42, p. 285 A woman at a point A on the shore of a circular lake with radius 2 mi wants to arrive at the point C diametrically opposite A in the shortest possible time. She can walk at the rate 4 mi/h and row a boat at 2 mi/h. How should she proceed?

Solution. We depict the situation as follows:



Let B be the point where she reaches the shore after rowing the boat. The quantity that needs to be minimized is (t_1+t_2), where t_1 is time necessary to go from A to B, and t_2 is time necessary to walk on the shore from B to C. We get $t_1 = \frac{AB}{v_1} = \frac{AB}{2}$ and $t_2 = \frac{\overset{\frown}{BC}}{4}$

The convenient way to set (t_1+t_2) as a function is to use as variable the angle $\theta = \measuredangle BAC$.

Then, as the triangle ABC is right angled at B, we have:

$$\cos\theta = \frac{AB}{AC} = \frac{AB}{4} \implies AB = 4\cdot\cos\theta$$

Then we notice that the angle $\measuredangle BOC$ has measure 2θ. Consequently the length of the the arc $\overset{\frown}{BC}$ is $2\theta\cdot \text{radius} = 4\theta$. (I used here the fact that the length of the circle is $2\pi \cdot$ radius, and corresponds to an interior angle of 2π radians. Consequently to an angle 2θ corresponds a length $2\theta \cdot$ radius.)

We are reduced to minimizing the function:

$$t(\theta) = t_1 + t_2 = \frac{AB}{2} + \frac{\overset{\frown}{BC}}{4} = 2\cos\theta + \theta,$$

where $\theta \in [0, \frac{\pi}{2}]$. (Note that $\theta = 0$ corresponds to only rowing and $\theta = \frac{\pi}{2}$ corresponds to only walking.

The graph of t is as below and if we get that the global minimum occurs when $\theta = \frac{\pi}{2}$. So the woman should walk all the way to C.

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> plot(2*cos(x)+x,x=0..Pi/2);
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