Exponential and Logarithm Functions

The Basics

If n and m are positive integers...

$$a^n = \underbrace{a \cdot a \cdot \cdots \cdot a}_{n} \qquad \text{(WeBWoRK: } a^\wedge n \text{ or } a**n\text{)}$$

Some identities:

Examples:

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$$(a^n)^{1/n} = a^{n*\frac{1}{n}} = a^1 = a$$

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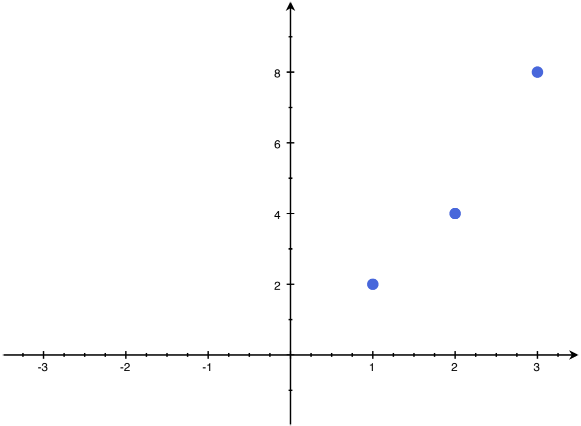
Example: $8^{5/3} = \left(\sqrt[3]{8}\right)^5 = 2^5 = 32$ or $8^{5/3} = \sqrt[3]{8^5} = \sqrt[3]{32{,}768} = 32$

What is a^x for all x?

If $a > 1$:

(e.g. $a = 2$)



$x = 1, 2, 3, \ldots$

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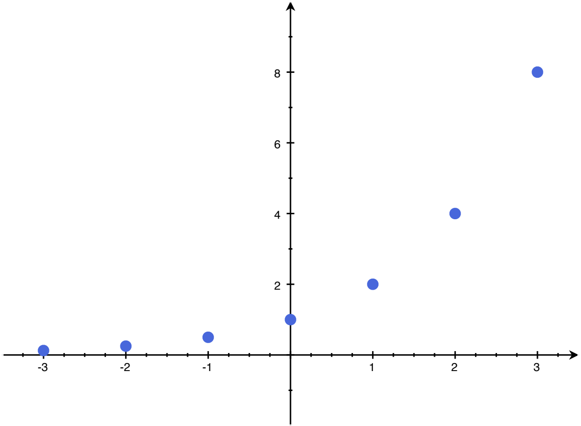

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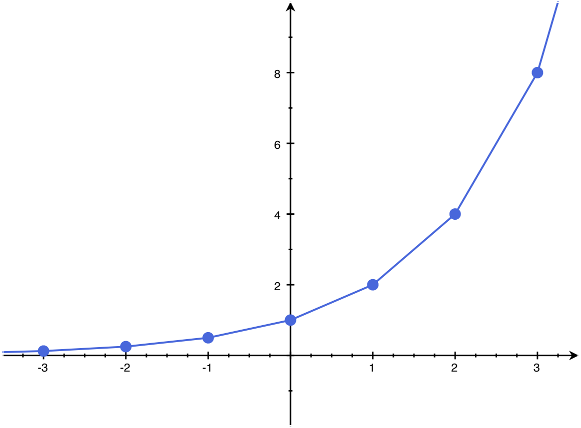

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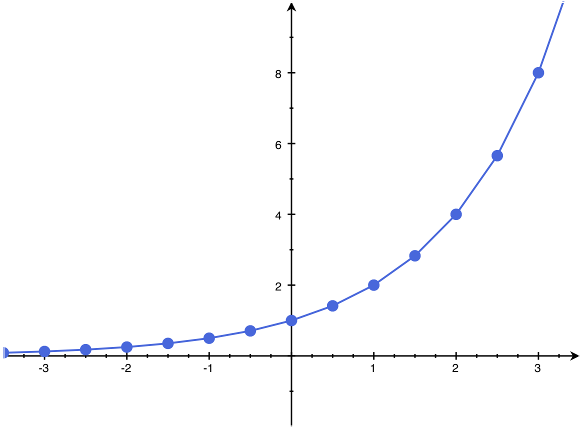

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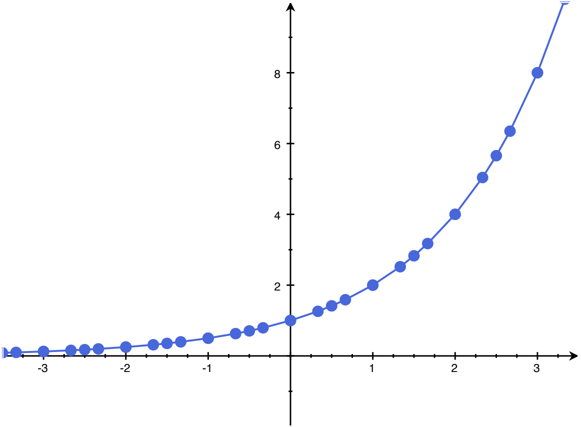

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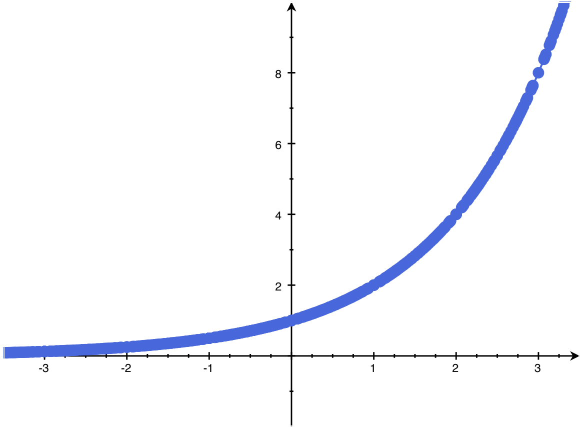

$x = n/2,\ n/3,\ \ldots,\ n/15$, for $n = 0, \pm 1, \pm 2, \pm 3, \ldots$

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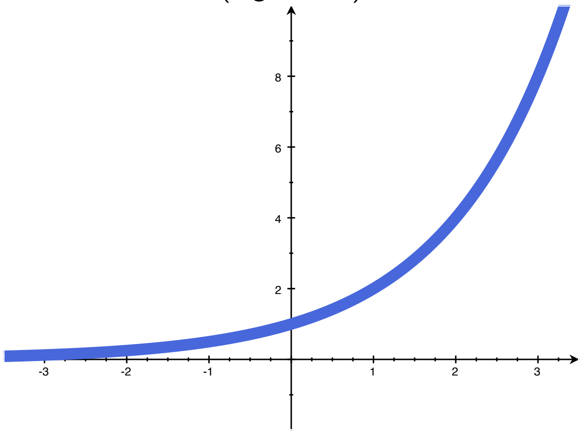

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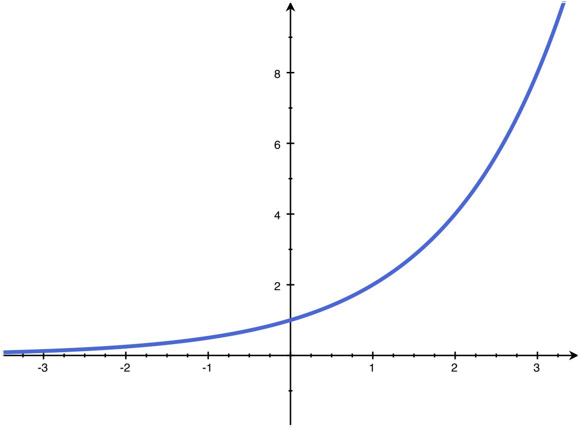

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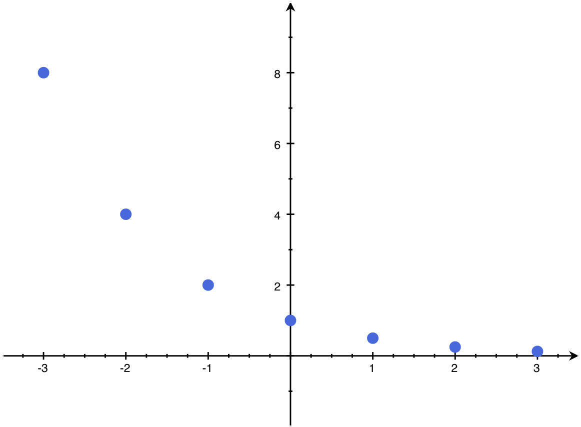

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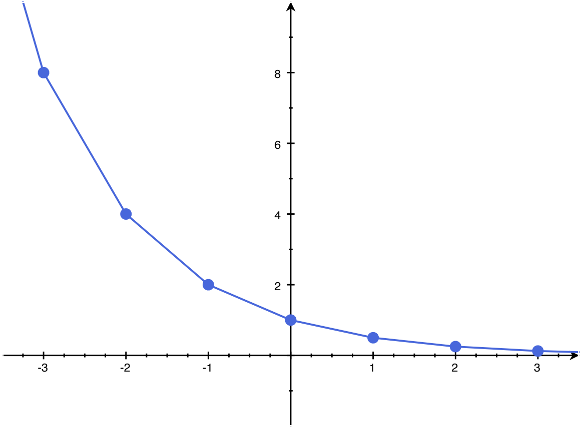

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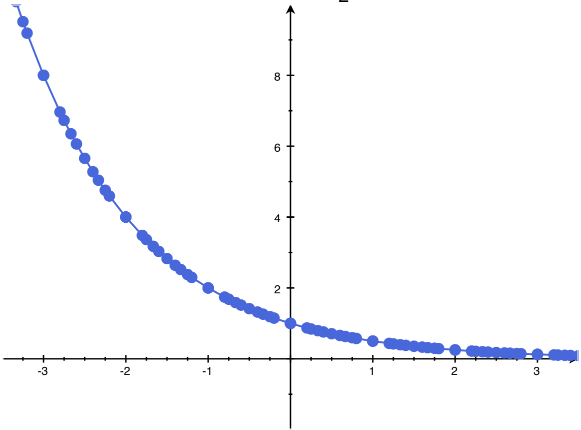

$x = n/2, n/3, n/4, n/5$, for $n = 0, \pm 1, \pm 2, \pm 3, \ldots$

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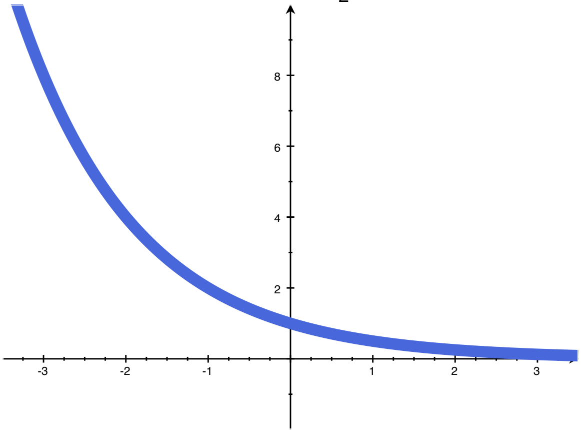

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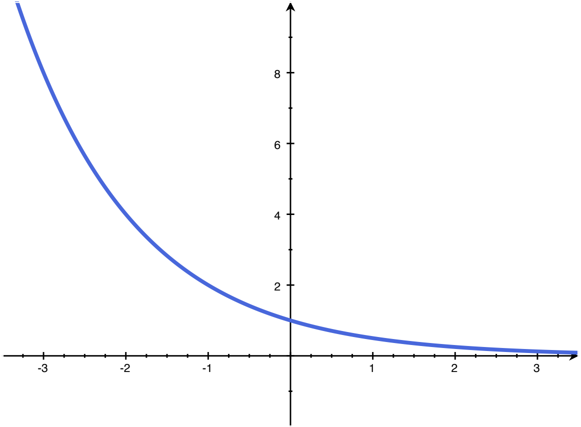

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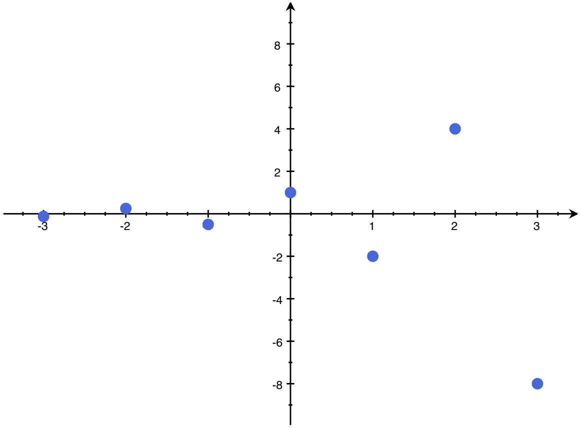

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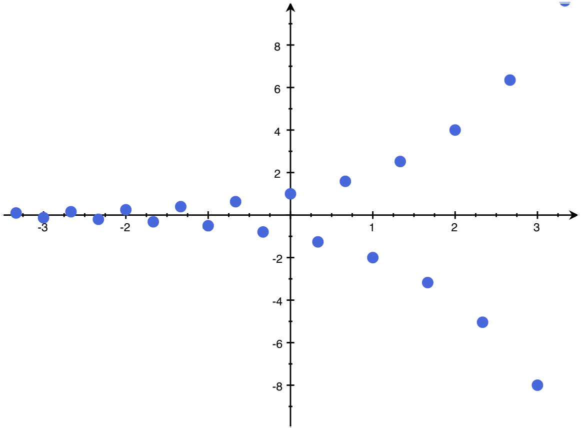

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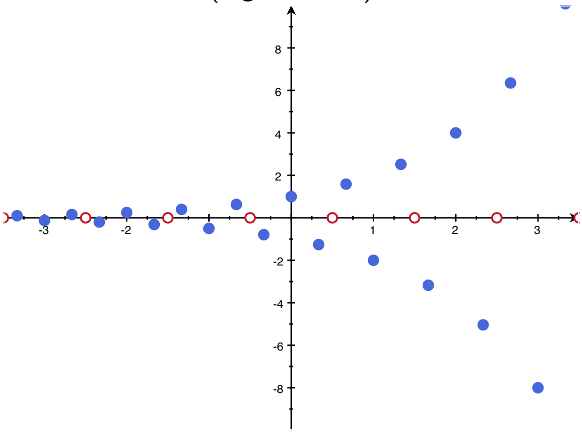

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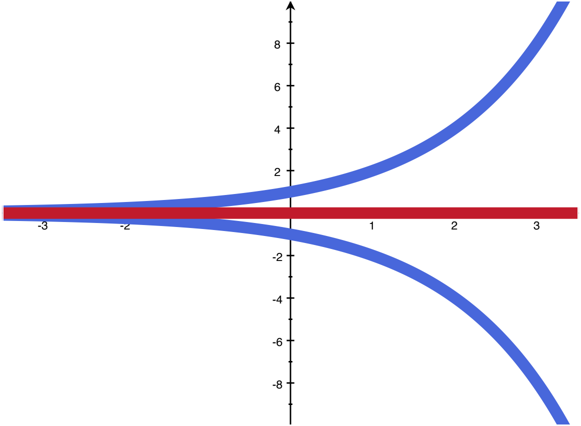

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OH NO!

The function a^x:

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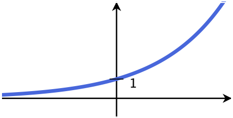

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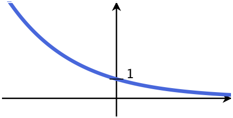

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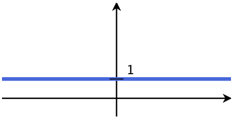

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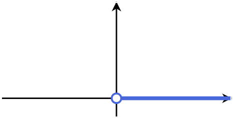


D: $(0, \infty)$, R: $\{0\}$

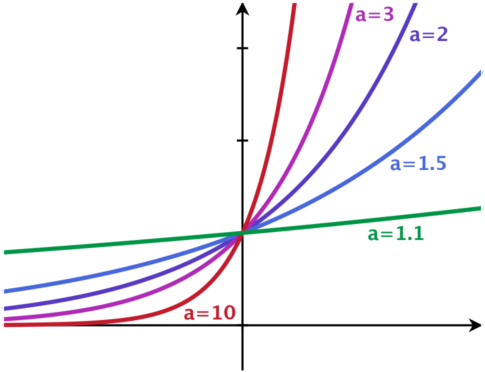
Properties:

$a^b * a^c = a^{b+c}$ $\quad (a^b)^c = a^{b*c}$ $\quad a^{-x} = 1/a^x$ $\quad a^c * b^c = (ab)^c$

Our favorite exponential function:

Look at how the function is increasing through the point $(0, 1)$:

$y = a^x$:



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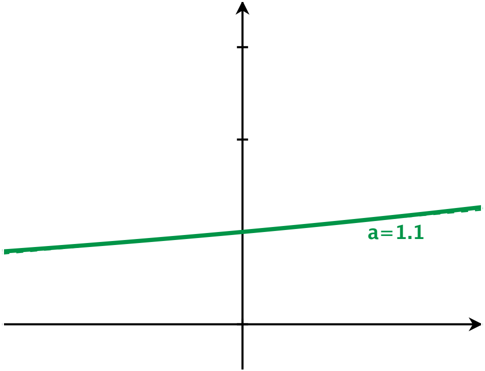

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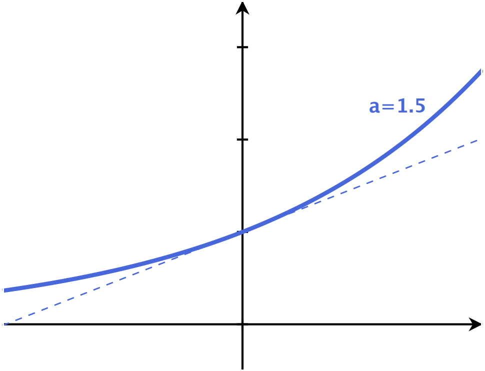

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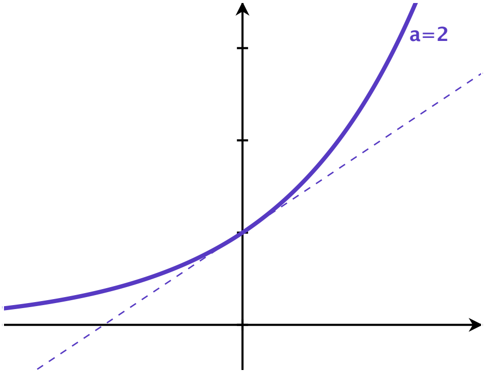

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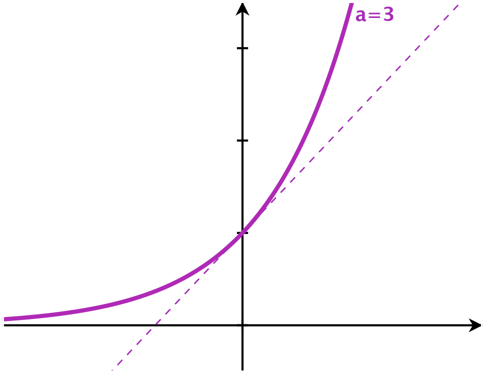

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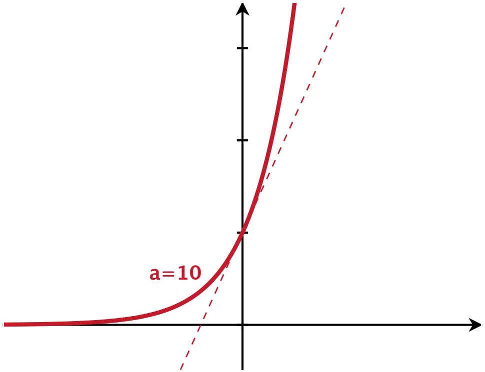

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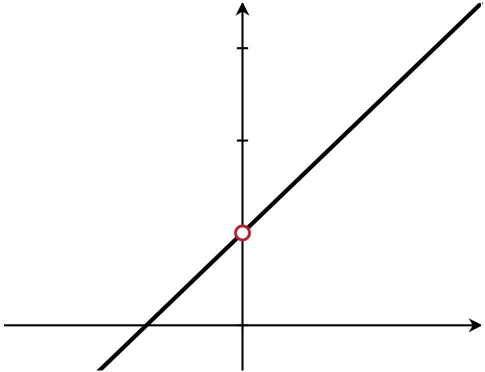

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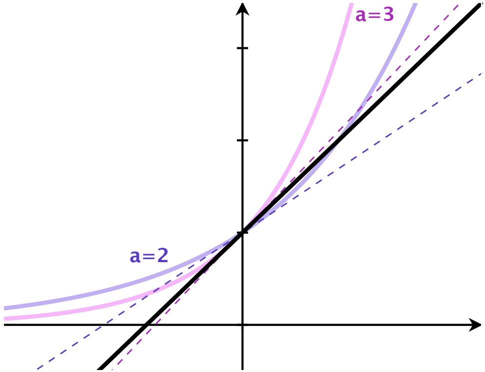

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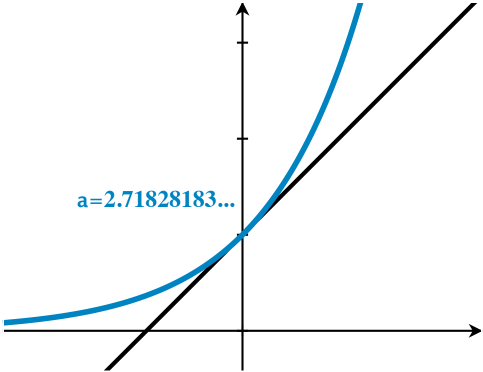

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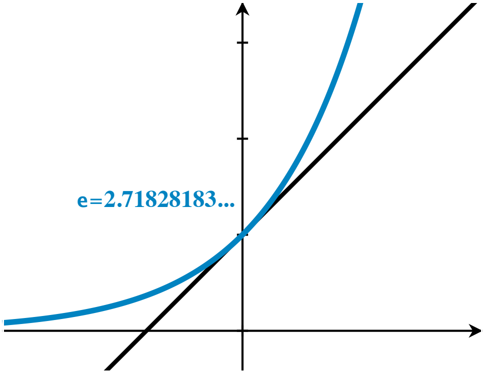

Q: Is there an exponential function whose slope at (0,1) is 1?

A: e^x is the exponential function whose slope at (0,1) is 1.

($e = 2.71828183\ldots$ is to calculus as $\pi = 3.14159265\ldots$ is to geometry)

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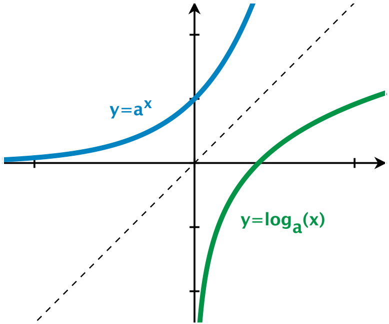
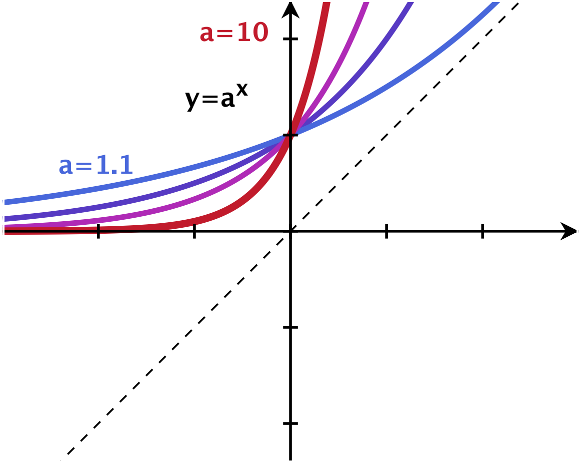


Properties of Logarithms



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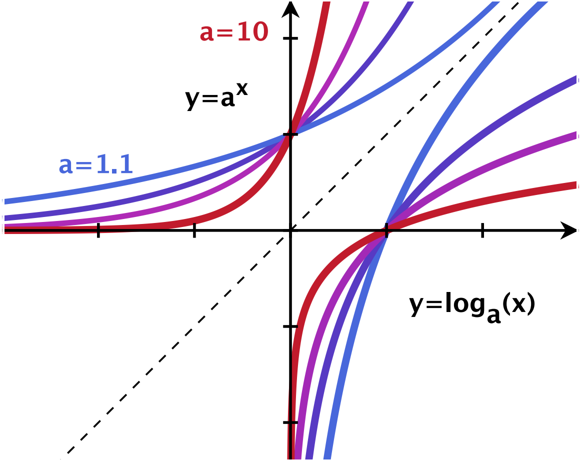

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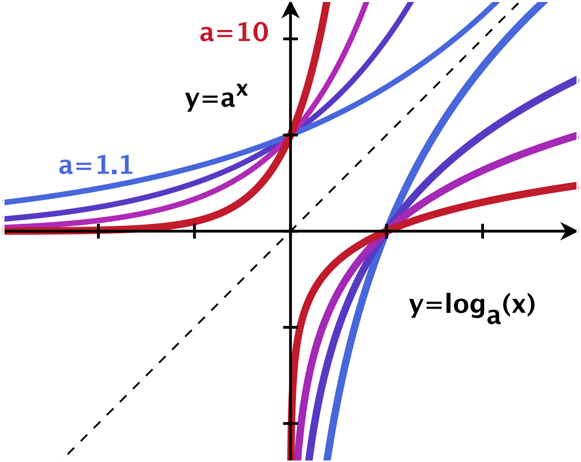
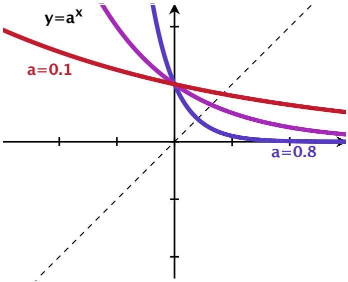
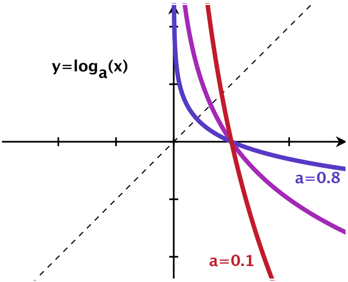


Domain: $(0, \infty)$ i.e. all $x > 0$ Range: $(-\infty, \infty)$ i.e. all x

Properties of Logarithms

$0 < a < 1$:





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Then $a^y = a^{\log_a(b)+\log_a(c)} = a^{\log_a(b)} a^{\log_a(c)} = b * c$.

So $y = \log_a(b * c)$ as well!

Lastly: $\dfrac{\log_a(b)}{\log_a(c)} = \log_c(b)$

4. let $y = c\log_a(b)$.

Then $a^y = a^{c\log_a(b)}$

$= \left(a^{\log_a(b)}\right)^c$

$= b^c$.

So $y = \log_a(b^c)$.

-or-

let $y = \log_a(b^c)$.

Then $a^y = b^c$

$= \left(a^{(\log_a(b))}\right)^c$

$= a^{c\cdot(\log_a(b))}$

So $y = c\log_a(b)$.

Change of base:

let $y = \log_a(c)\cdot\log_c(b)$

Then $a^y = a^{(\log_a(c))\cdot(\log_c(b))}$

$= \left(a^{\log_a(c)}\right)^{\log_c(b)}$

$= (c)^{\log_c(b)}$

$= b$. (so $a^y = b$)

So $y = \log_a(b)$

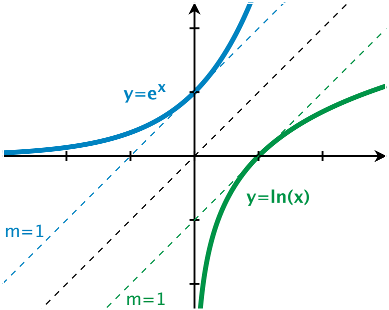
Thus $\log_a(b) = \log_a(c)\cdot\log_c(b)$

so $\log_a(b)/\log_a(c) = \log_c(b)$.

Favorite logarithmic function

Remember: $y = e^x$ is the function whose slope through the point (0,1) is 1.
The *inverse* to $y = e^x$ is the *natural log*:

$$\ln(x) = \log_e(x)$$



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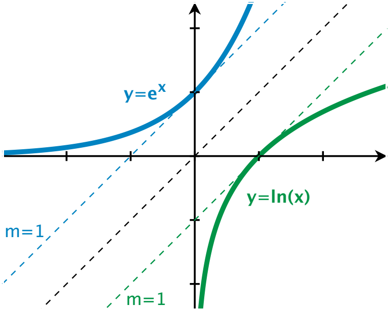

We will often use the facts that $e^{\ln(x)} = x$ (for $x > 0$) and $\ln(e^x) = x$ (for all x)

Two super useful facts:

Explain why:

(1) $\log_a(b) = \ln(b)/\ln(a)$

(2) $a^b = e^{b\ln(a)}$ [hint: start by rewriting $b\ln(a)$, and use the fact that $e^{\ln(x)} = x$]

Two super useful facts:

Explain why:

(1) $\log_a(b) = \ln(b)/\ln(a)$

Since $\ln(b) = \log_e(b)$ and $\ln(a) = \log_e(a)$, we have

$$\frac{\ln(b)}{\ln(a)} = \frac{\log_e(b)}{\log_e(a)} = \log_a(b)$$

(2) $a^b = e^{b\ln(a)}$ [hint: start by rewriting $b\ln(a)$, and use the fact that $e^{\ln(x)} = x$]

Since $b\ln(a) = \ln(a^b)$ and $e^{\ln(x)} = x$, we have

$$e^{b\ln(a)} = e^{\ln(a^b)} = a^b$$

Examples:

(1) Condense the logarithmic expressions

$$\frac{1}{2}\ln(x)+3\ln(x+1) \qquad 2\ln(x+5)-\ln(x) \qquad \frac{1}{3}(\log_3(x)-\log_3(x+1))$$

(2) Solve the following expressions for x:

$$e^{-x^2} = e^{-3x-4} \qquad 3(2^x) = 24$$

$$2(e^{3x-5}) - 5 = 11 \qquad \ln(3x+1) - \ln(5) = \ln(2x)$$

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$$\frac{1}{2}\ln(x)+3\ln(x+1) \qquad 2\ln(x+5)-\ln(x) \qquad \frac{1}{3}(\log_3(x)-\log_3(x+1))$$

$$\boxed{\ln(\sqrt{x}(x+1)^3)} \qquad \boxed{\ln\left(\frac{(x+5)^2}{x}\right)} \qquad \boxed{\log_3\left(\left(\frac{x}{x+1}\right)^{1/3}\right)}$$

(2) Solve the following expressions for x:

$$e^{-x^2} = e^{-3x-4} \qquad 3(2^x) = 24$$

$$\boxed{x = -1, 4} \qquad \boxed{x = 3}$$

$$2(e^{3x-5}) - 5 = 11 \qquad \ln(3x+1) - \ln(5) = \ln(2x)$$

$$\boxed{x = \frac{\ln(8)+5}{3}} \qquad \boxed{x = \frac{1}{7}}$$

$$\frac{1}{2}\ln(x) + 3\ln(x+1) = \ln(x^{1/2}) + \ln((x+1)^3)$$

$$= \ln\left(x^{1/2}(x+1)^3\right)$$

$$2\ln(x+5) - \ln(x) = \ln((x+5)^2) + \ln(x^{-1})$$

$$= \ln\left((x+5)^2 \cdot x^{-1}\right) = \ln\left(\frac{(x+5)^2}{x}\right)$$

$$\frac{1}{3}\left(\log_3(x) - \log_3(x+1)\right) = \frac{1}{3}\left(\log_3\left(x(x+1)^{-1}\right)\right)$$

$$= \log_3\left(\sqrt[3]{\frac{x}{x+1}}\right)$$

If $e^{-x^2} = e^{-3x-4}$, then $-x^2 = -3x-4$,

(take ln(–) both sides)

so $x^2 - 3x - 4 = 0$

$(x-4)(x+1) =$

so $x = 4$ or -1

If $3(2^x) = 24$, then $2^x = 8$,

so $\boxed{x = 3}$

If $2(e^{3x-5}) - 5 = 11$,

then $e^{3x-5} = \dfrac{11+5}{2} = 8$

so $3x - 5 = \ln(8)$

so $x = \boxed{\dfrac{\ln(8)+5}{3}}$

If $\ln(3x+1) - \ln(5) = \ln(2x)$

$$e^{\left(\ln\left(\frac{3x+1}{5}\right)\right)} = e^{\left(\ln(2x)\right)}$$

$$\frac{3x+1}{5} = 2x$$

$$3x+1 = 10x$$

$$1 = 7x$$

$$\boxed{x = \frac{1}{7}}$$

$$\left(\ln(3x+1) - \ln(5) = \ln(3x+1) + \ln(5^{-1})\right.$$

$$= \ln\left((3x+1)5^{-1}\right) = \ln\left(\frac{3x+1}{5}\right)\Bigg)$$

$$\ln(a) - \ln(b) = \ln(a) + \ln(b^{-1}) = \ln(a/b)$$