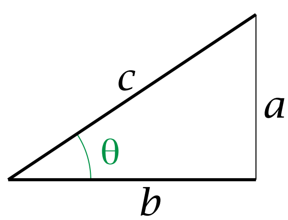
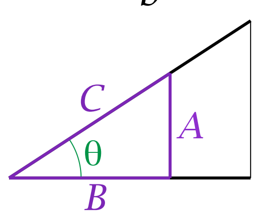
Trigonometric functions

Step one: similar triangles





Two similar triangles have the same set of angles, and have the properties that

$$\frac{A}{B} = \frac{a}{b}, \quad \frac{B}{C} = \frac{b}{c}, \text{ and } \frac{A}{C} = \frac{a}{c}.$$

Define

$$\cos(\theta) = \frac{b}{c} \quad \text{and} \quad \sin(\theta) = \frac{a}{c}.$$

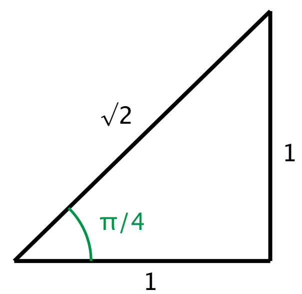
Then let

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{a}{b}, \qquad \csc(\theta) = \frac{1}{\sin(\theta)} = \frac{c}{a},$$

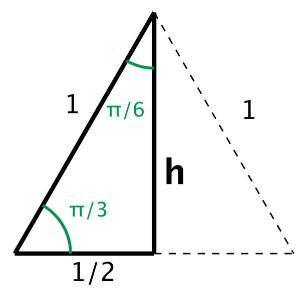
$$\sec(\theta) = \frac{1}{\cos(\theta)} = \frac{c}{b}, \qquad \cot(\theta) = \frac{1}{\tan(\theta)} = \frac{b}{a}.$$

Easy angles:

isosceles right triangle:



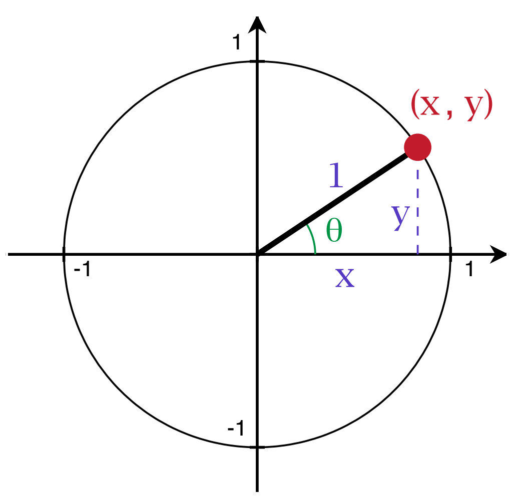
equilateral triangle cut in half:



$h = \sqrt{1-(1/2)^2} = \sqrt{3}/2$

	$\cos(\theta)$	$\sin(\theta)$	$\tan(\theta)$	$\sec(\theta)$	$\csc(\theta)$	$\cot(\theta)$
$\pi/4$						
$\pi/3$						
$\pi/6$						

Step two: the unit circle



For $0 < \theta < \frac{\pi}{2}$...

$$\cos(\theta) = \frac{x}{1} = x$$

$$\sin(\theta) = \frac{y}{1} = y$$

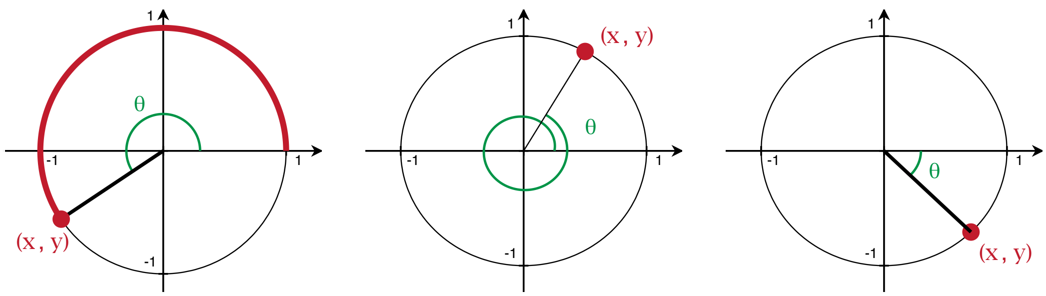
Use this idea to extend trig functions to any θ...

Define

$$\cos(\theta) = x \qquad \sin(\theta) = y,$$

there θ is defined by...

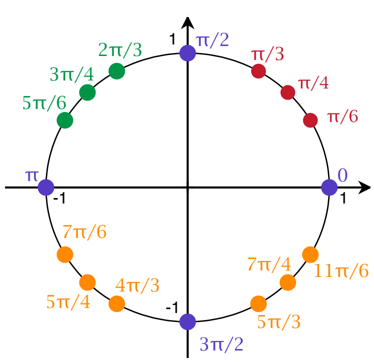
$0 \le \theta \le 2\pi$ | all $\theta \ge 0$ | $\theta < 0$



Sidebar: In calculus, radians are king. Where do they come from?
Circumference of a unit circle: 2π
Arclength of a wedge with angle θ*:*

$$\frac{\theta}{180^\circ} * 2\pi \quad \textit{(if in degrees)} \qquad \textit{or} \qquad \frac{\theta}{2\pi} * 2\pi = \boxed{\theta} \quad \textit{(if in radians)}$$

Reading off of the unit circle



$$\cos(\pi - \theta) = -\cos(\theta) \qquad \sin(\pi - \theta) = \sin(\theta)$$

$$\cos(-\theta) = \cos(\theta) \qquad \sin(-\theta) = -\sin(\theta)$$

$$\cos(2\pi n + \theta) = \cos(\theta) \qquad \sin(2\pi n + \theta) = \sin(\theta)$$

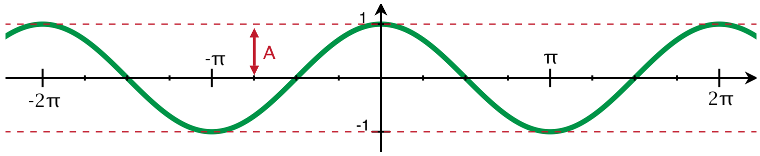
$$x^2 + y^2 = 1 \implies \cos^2(\theta) + \sin^2(\theta) = 1$$

	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\cos(\theta)$							
$\sin(\theta)$							

	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$
$\cos(\theta)$									
$\sin(\theta)$									

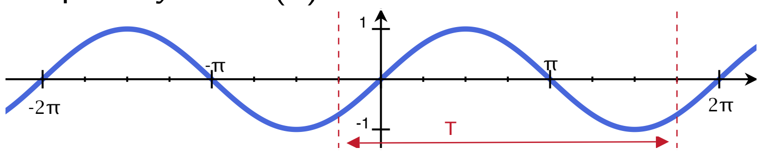
Plotting on the θ-y axis

Graph of $y = \cos(\theta)$:



A = Amplitude = $\frac{1}{2}$ length of the range = 1
T=Period = time to repeat = 2π

Graph of $y = \sin(\theta)$:



A=Amplitude = $\frac{1}{2}$ length of the range = 1
T=Period = time to repeat = 2π

Trig identities to know and love:

Even/odd:

$$\cos(-\theta) = \cos(\theta) \quad \text{(even)} \qquad \sin(-\theta) = -\sin(\theta) \quad \text{(odd)}$$

Pythagorean identity:

$$\cos^2(\theta) + \sin^2(\theta) = 1$$

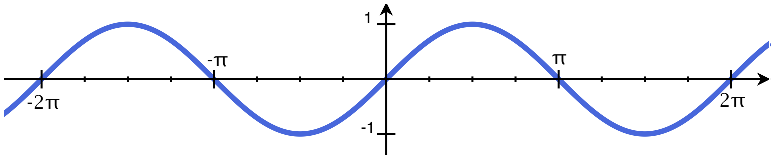
Angle addition:

$$\cos(\theta + \phi) = \cos(\theta)\cos(\phi) - \sin(\theta)\sin(\phi)$$

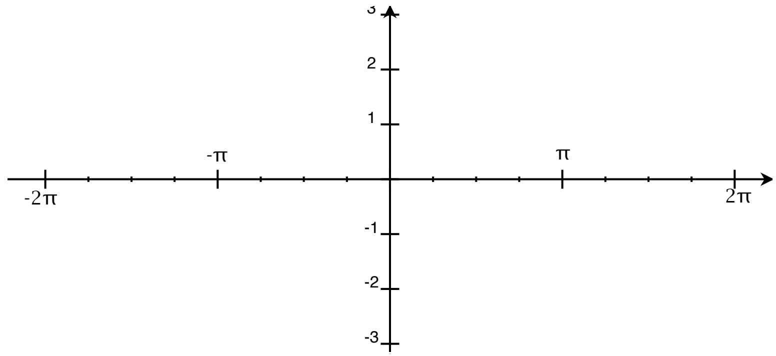
$$\sin(\theta + \phi) = \sin(\theta)\cos(\phi) + \cos(\theta)\sin(\phi)$$

(in particular $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$ *and* $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$*)*

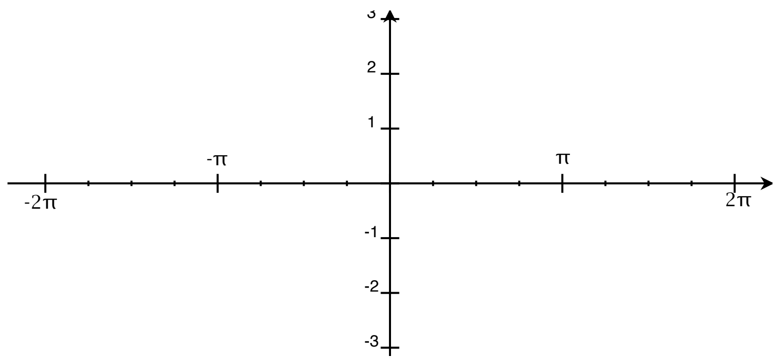
Transform the graph of $\sin(\theta)$ into the graph of $2\sin(\frac{1}{2}\theta + \pi/6) - 1$:



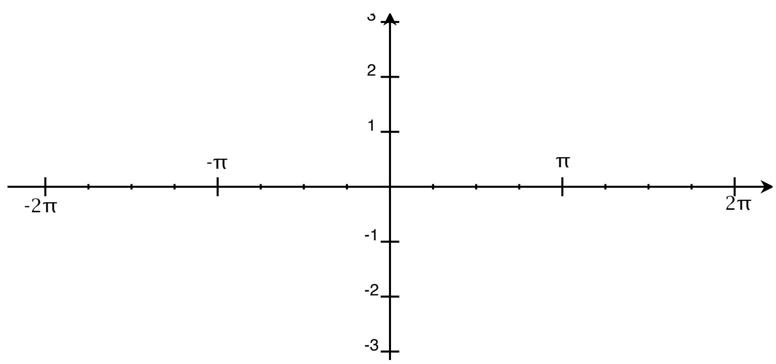
$\xrightarrow{\sin\left(\frac{1}{2}\theta\right)}$



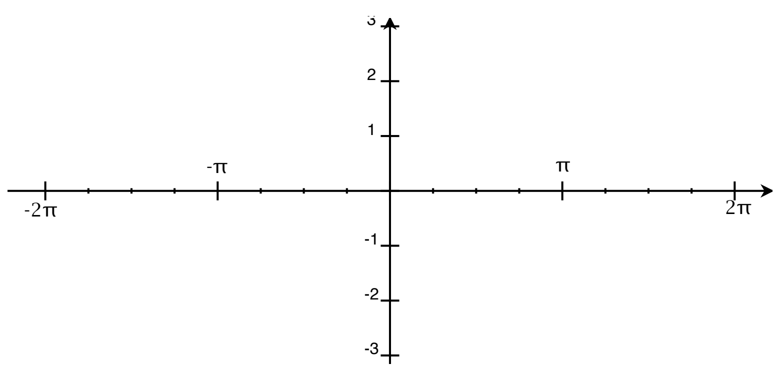
$\xrightarrow{\sin\left(\frac{1}{2}(\theta+\frac{\pi}{3})\right)}$



$\xrightarrow{2\sin\left(\frac{1}{2}(\theta+\frac{\pi}{3})\right)}$



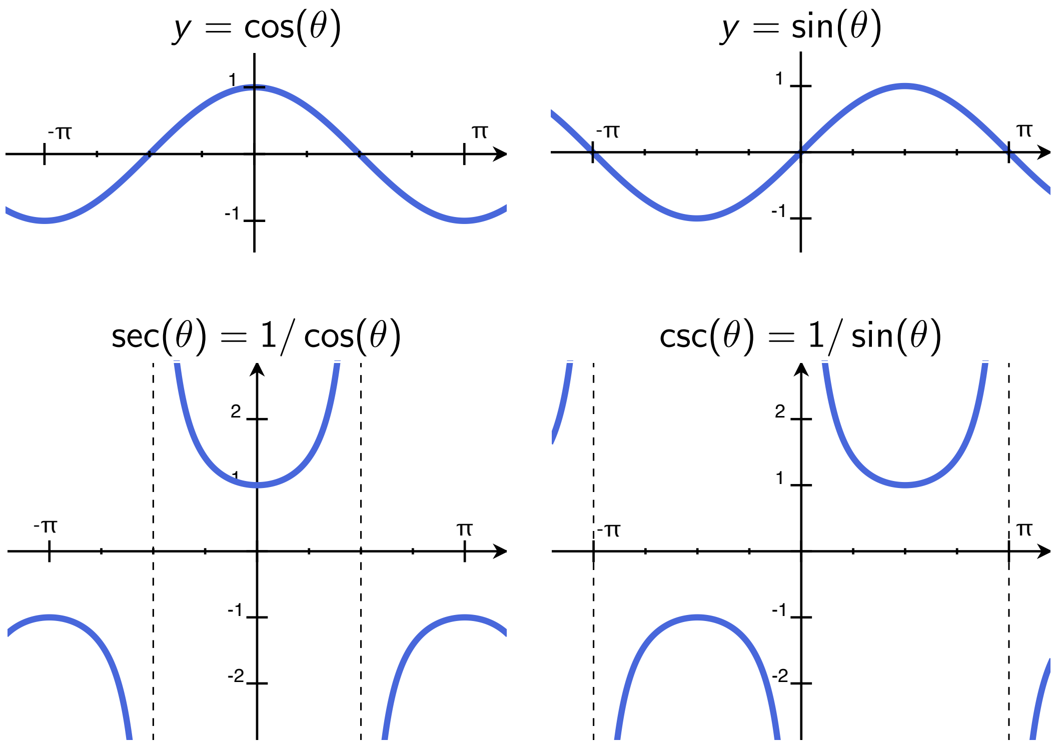
$\xrightarrow{2\sin\left(\frac{1}{2}(\theta+\frac{\pi}{3})\right)-1}$



What is the amplitude of $2\sin\left(\frac{1}{2}\theta + \frac{\pi}{6}\right) - 1$?

What is the period?

Other trig functions



Other trig functions