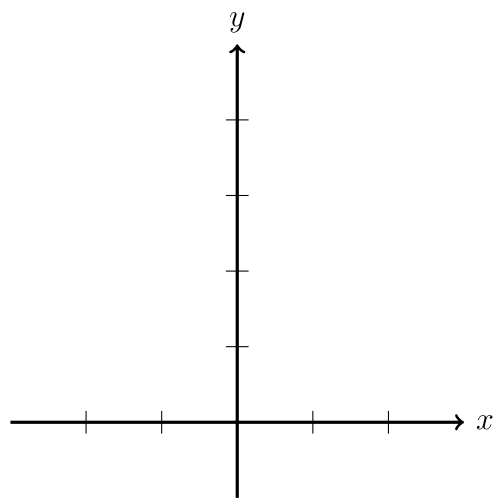
Math 3 - Day 1 - WARMUP

Calculate the given functions at the given x-values, and then plot the corresponding points.

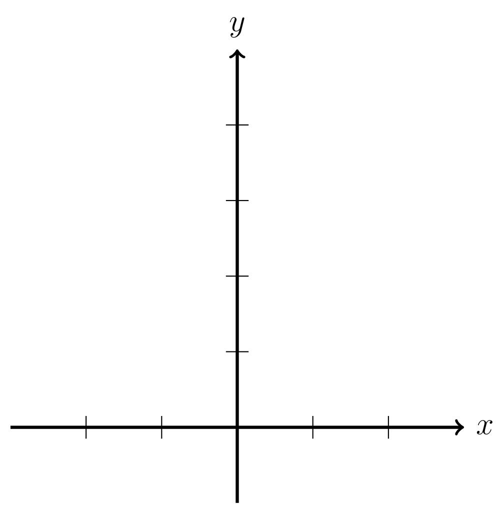
$f(x) = x^2$

x	$f(x)$
-2	
-1	
0	
1	
2	



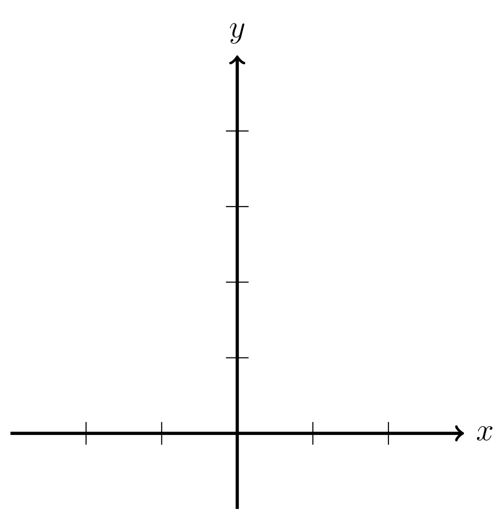
$f(x) = (x+1)^2$

x	$f(x)$
-2	
-1	
0	
1	



$f(x) = x^2 + 1$

x	$f(x)$
-2	
-1	
0	
1	
2	



Welcome to Math 3!

http://www.math.dartmouth.edu/~m3f13/

Me:

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314 Kemeny Hall
zajj.b.daugherty@dartmouth.edu
Office hours: Wednesday 2-3, Thursday 10-12, or by appointment
Extra credit: Your first visit to office hours after this week

Tutorials:

7:00 to 9:00 pm on Sunday, Tuesday, and Thursday
Begins tomorrow in 008 Kemeny.

Highlights:

Homework: 15% of your grade
Tips:
All done on WeBWorK – due at 8:00 a.m two classes later (answers up at 12pm)
When possible, use exact answers like "2^6 − 1" instead of "63".
Wrong submissions don't count against you, unless otherwise stated.

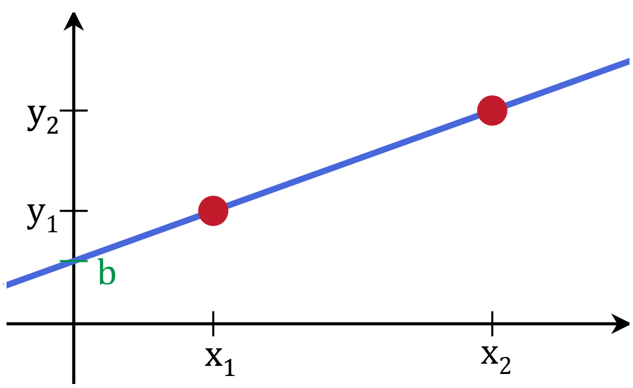
Pro: as many tries as you need
Con: computers are fussy

Midterms: 25% × 2
Part multiple choice, part long-answer.
Dates:
Midterm 1: Wed 10/16, 7–9 pm
Midterm 2: Wed 11/6, 7–9 pm

Final: 35%
All multiple choice. Cumulative.
Date: 11/22, 11:30 am

Functions and their graphs

Simplest functions: Lines!



Two points define a line!

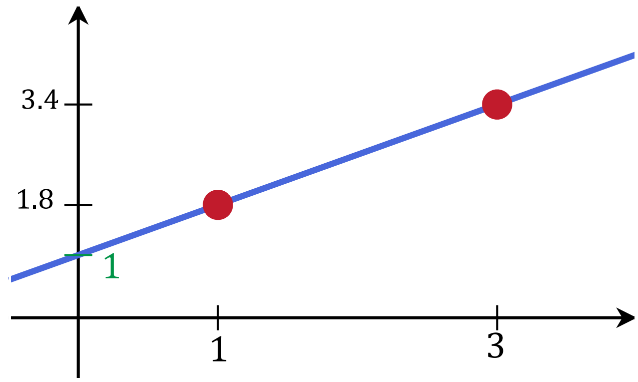
Slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ (rise/run)

Point-slope form: $y - y_1 = m(x - x_1)$ (good for writing down lines)

Slope-intercept form: $y = mx + b$ (good for graphing)

General form: $Ax + By + C = 0$ (accounts for ∞ slope)

Simplest functions: Lines!



Example:

Slope: $m = \frac{3.4 - 1.8}{3 - 1} = 0.8$ (rise/run)

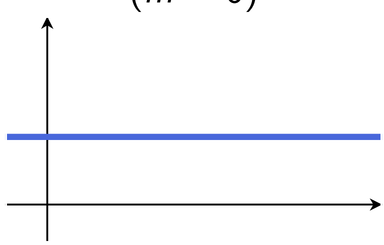
Point-slope form: $y - 1.8 = 0.8 * (x - 1)$ (good for writing down lines)

Slope-intercept form: $y = 0.8 * x + 1$ (good for graphing)

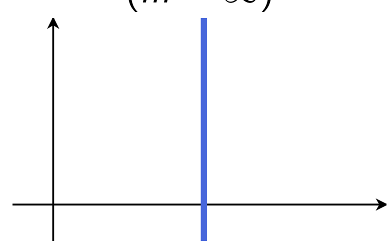
General form: $0.8 * x + y - 1 = 0$ (accounts for ∞ slope)

Lines: Special cases

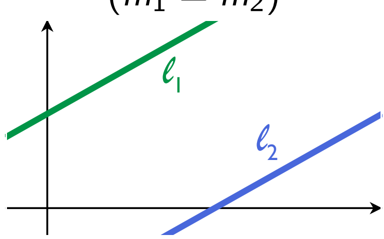
Constant functions
($m = 0$)



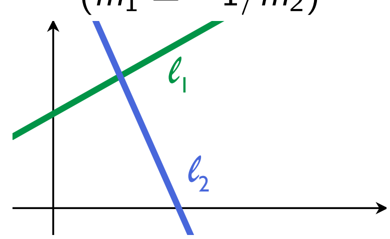
Vertical lines
($m = \infty$)



Parallel lines
($m_1 = m_2$)



Perpendicular lines
($m_1 = -1/m_2$)



Other good functions to know: polynomials.

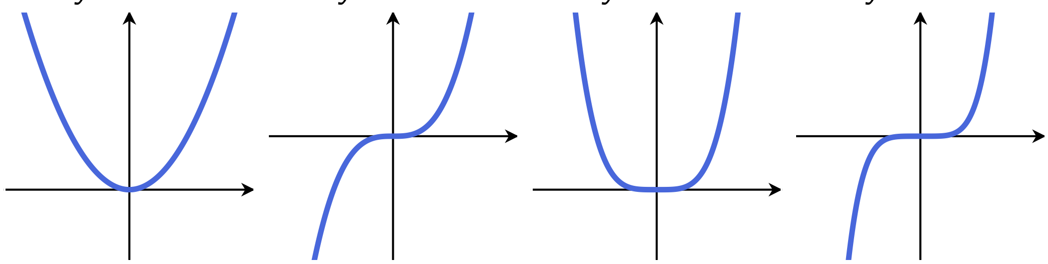
$$y = a_0 + a_1 x + \cdots + a_n x^n$$

(n is the *degree*)

The basics (know these graphs!)

$n = 0$:	$n = 1$:	$n = 2$:
constants	lines	parabolas

$y = x^2$ $y = x^3$ $y = x^4$ $y = x^5$

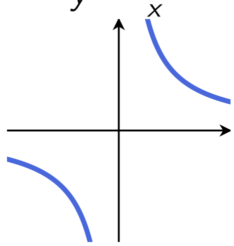


Other good functions to know: rationals.

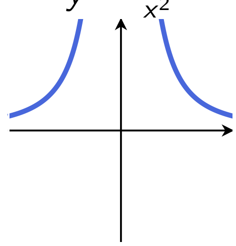
$$y = \frac{a_0 + a_1 x + \cdots + a_n x^n}{b_0 + b_1 x + \cdots + b_m x^m}$$

The basics (know these graphs!)

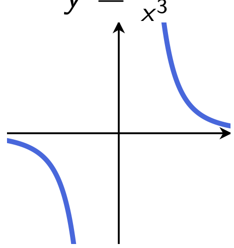
$y = \frac{1}{x}$



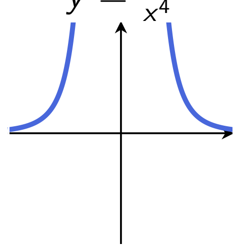
$y = \frac{1}{x^2}$



$y = \frac{1}{x^3}$



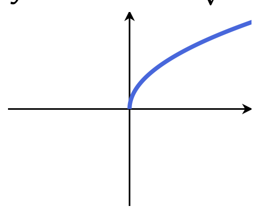
$y = \frac{1}{x^4}$



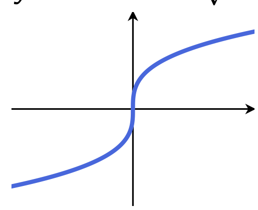
Other powers: $y = x^a$.

The basics (know these graphs!)

$y = x^{1/2} = \sqrt{x}$

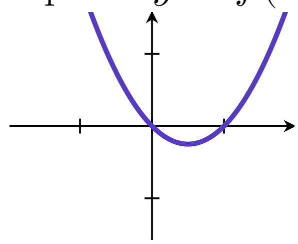


$y = x^{1/3} = \sqrt[3]{x}$

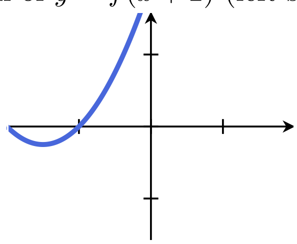


New functions from old

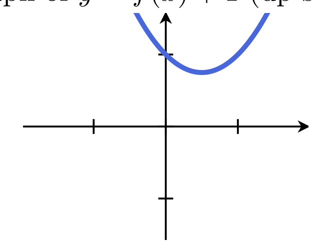
Graph of $y = f(x)$:



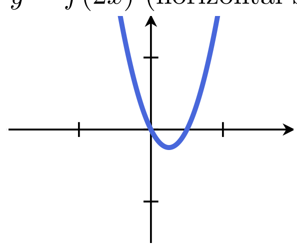
Graph of $y = f(x+2)$ (left shift) :



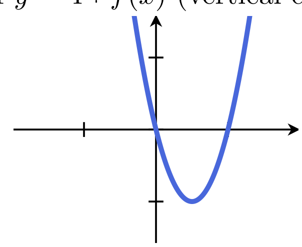
Graph of $y = f(x) + 1$ (up shift):



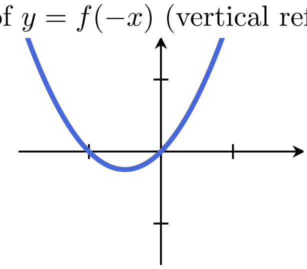
Graph of $y = f(2x)$ (horizontal squeeze) :



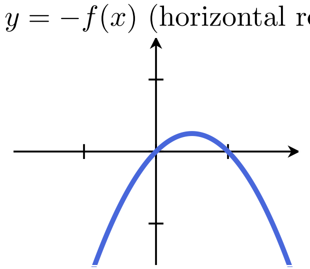
Graph of $y = 4 * f(x)$ (vertical dialation):



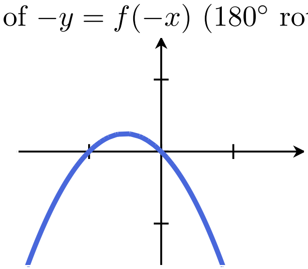
Graph of $y = f(-x)$ (vertical reflection) :



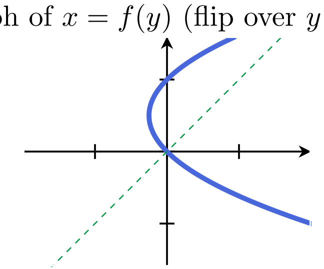
Graph of $y = -f(x)$ (horizontal reflection):



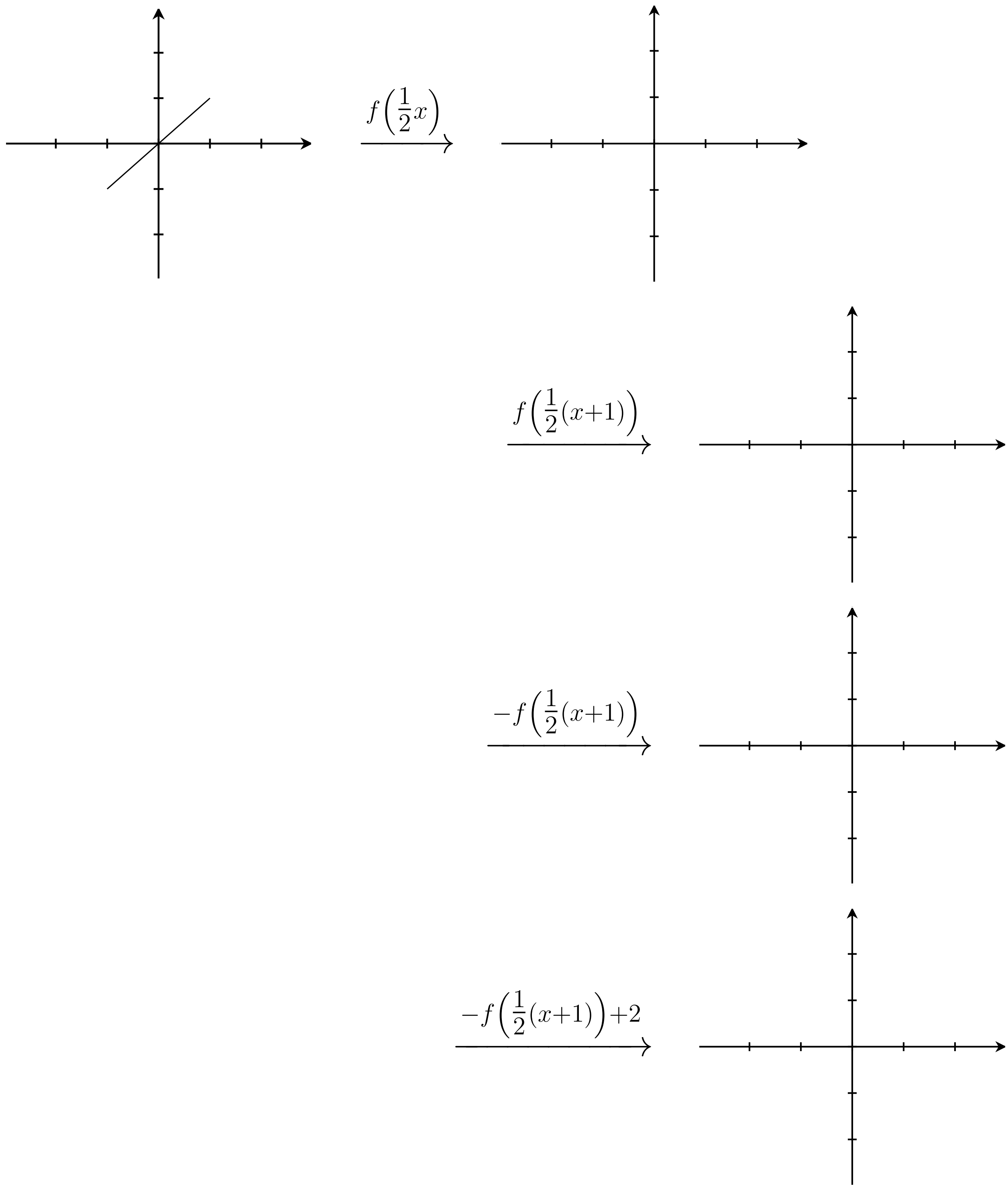
Graph of $-y = f(-x)$ (180° rotation) :



Graph of $x = f(y)$ (flip over $y = x$):



Ex: Transform the graph of $f(x)$ into the graph of $-f\left(\frac{1}{2}(x+1)\right)+2$:

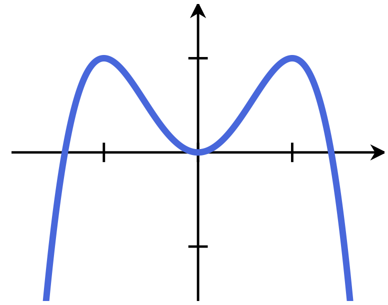


The *domain* of a function f is the set of x over which $f(x)$ is defined.

The *range* of a function f is the set of y which satisfy $y = f(x)$ for some x.

Symmetries

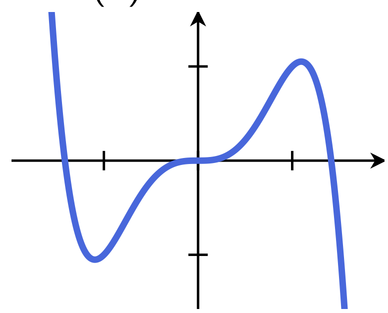
ex: $f(x) = 2x^2 - x^4$



A function $f(x)$ is *even* if it satisfies

$$f(-x) = f(x)$$

ex: $f(x) = 2x^3 - x^5$

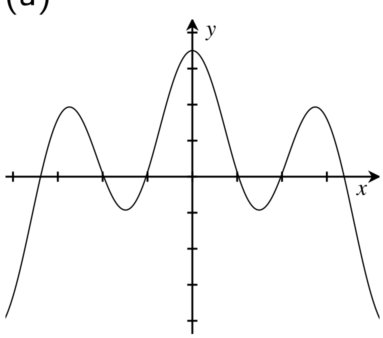


A function $f(x)$ is *odd* if it satisfies

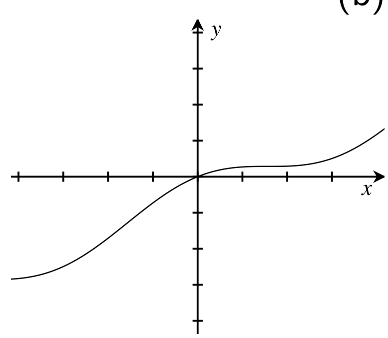
$$f(-x) = -f(x)$$

Examples: Even, odd, or neither?

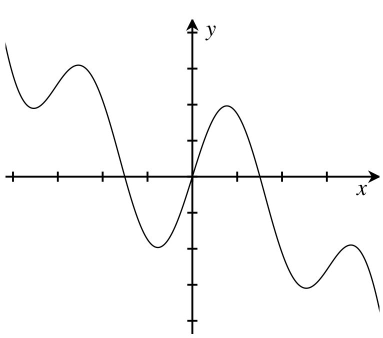
(a)



(b)



(c)

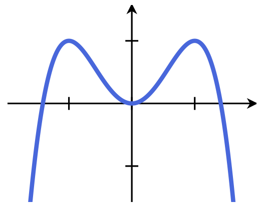


(d) $f(x) = \dfrac{x^3 + x}{x + \frac{1}{x}}$

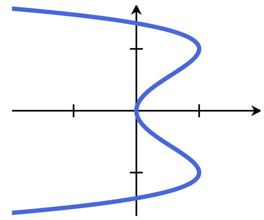
(for this one:
actually plug in $-x$
and see what happens
algebraically)

A graph is a graph of a *function* if for every x in its domain, there is exactly one y on the graph which is mapped to by that x:

Function:

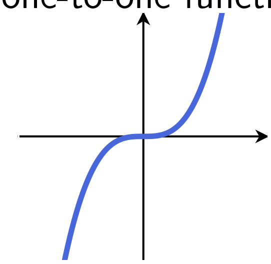


Not a function:



A function is additionally *one-to-one* if for every y, there is at most one x which maps to that y.

A one-to-one function:



Inverse functions

Let f be a one-to-one function.
If g is a function satisfying

$$f(g(x)) = g(f(x)) = x$$

then g is the *inverse function* of f. Write $g(x) = f^{-1}(x)$.

Example: If $f(x) = x^3$, then $f^{-1}(x) = \sqrt[3]{x}$

To calculate $f^{-1}(x)$, set $f(y) = x$ and solve for y. Then $y = f^{-1}(x)$.

Example: If $f(x) = \frac{2x}{x-1}$,

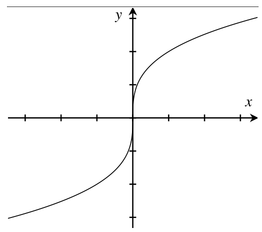
$$\text{solve } x = \frac{2y}{y-1} \text{ for } y \text{ to get } y = \frac{x}{x-2}.$$

So $f^{-1}(x) = \frac{x}{x-2}$.

To get the graph of $f^{-1}(x)$, flip the graph of $f(x)$ over the line $y = x$.

Pair up graphs with their inverses:

A



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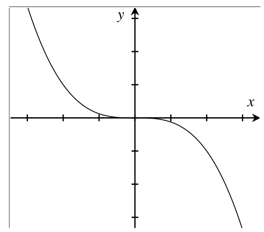

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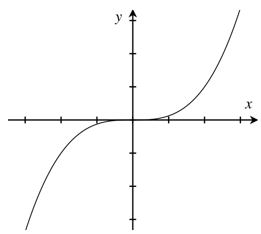
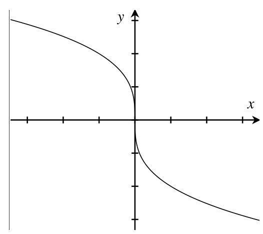


D



Worksheet – Functions – Math 3 – Sep 16, 2013

One way to build functions is by composition, i.e. plugging one function into another. If $f(x)$ and $g(x)$ are functions, then for whatever x for which $g(x)$ is in the domain of $f(x)$, then we can write

$$(f \circ g)(x) = f(g(x)).$$

For example, if $f(x) = \frac{x+1}{3x-2}$ and $g(x) = \sqrt{x}$, then

$$(f \circ g)(x) = \frac{\sqrt{x}+1}{3\sqrt{x}-2} \quad (\text{for } x \neq \pm(2/3)^2)$$

and

$$(g \circ f)(x) = \sqrt{\frac{x+1}{3x-2}} \quad (\text{whenever } \frac{x+1}{3x-2} \geq 0).$$

1. Let $f(x) = \dfrac{x+1}{3x-2}$ and $g(x) = \dfrac{1}{x}$.

   (a) Calculate $(f \circ g)(x)$ and $(g \circ f)(x)$.

   (b) What is the domain of $(g \circ f)(x)$?
   [hint: Careful! The domain of $(g \circ f)(x)$ is the set of x's which satisfy *both* (1) $f(x)$ exists, and (2) $(g \circ f)(x)$ exists.]

2. Let $f = \frac{x+1}{3x-2}$

   (a) Calculate $f^{-1}(x)$.

   (b) Check your answer to #1 by explicitly calculating $(f \circ f^{-1})(x)$ and $(f^{-1} \circ f)(x)$ (you should get x both times).

(c) If $(f \circ g)(x) = x + 2$, what is $g(x)$?
[hint: since $(f \circ g)(x) = f(g(x)) = x + 2$, we know

$$g(x) = f^{-1}(f(g(x)) = f^{-1}(x+2).]$$

Answers:

1. (a) $(f \circ g)(x) = \frac{\frac{1}{x}+1}{3\frac{1}{x}-2}$, $(g \circ f)(x) = \frac{3x-2}{x+1}$

   (b) All $x \neq 2/3, -1$, i.e. $(-\infty, -1) \cup (-1, 2/3) \cup (2/3, \infty)$

2. (a) $f^{-1}(x) = \frac{2x+1}{3x-1}$

   (b) (calculation)

   (c) $g(x) = \frac{2x+5}{3x+5}$