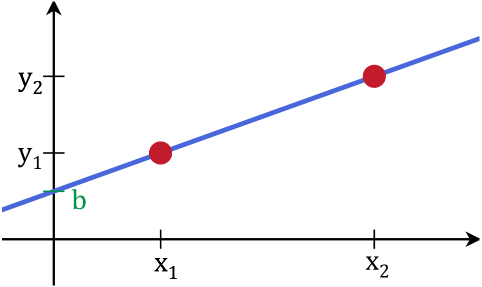
Functions and their graphs

Simplest functions: Lines!



Two points define a line!

Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$ (rise/run)

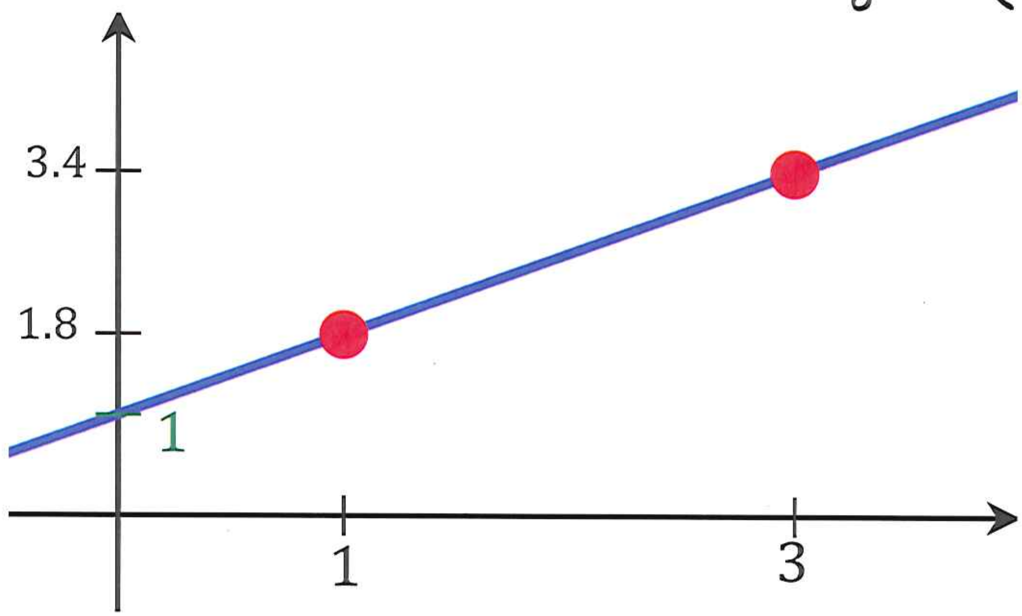
Point-slope form: $y - y_1 = m(x - x_1)$ (good for writing down lines)

Slope-intercept form: $y = mx + b$ (good for graphing)

General form: $Ax + By + C = 0$ (accounts for ∞ slope)

Simplest functions: Lines!

Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$



unless vertical!
$x = c$
Domain: $\{c\}$
Range: $(-\infty, \infty$

Example:

Slope: $m = \dfrac{3.4 - 1.8}{3 - 1} = 0.8$ (rise/run)

Point-slope form: $y - 1.8 = 0.8 * (x - 1)$ (good for writing down lines)

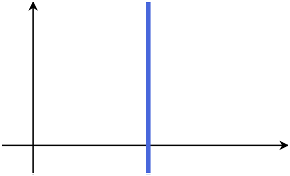
Slope-intercept form: $y = 0.8 * x + 1$ (good for graphing)

General form: $-0.8 * x + y - 1 = 0$ (accounts for ∞ slope)

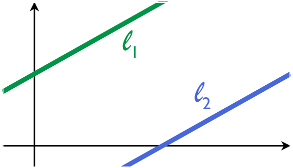
Lines: Special cases

Constant functions
($m = 0$)

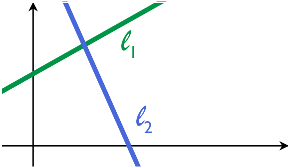
Vertical lines
($m = \infty$)



Parallel lines
($m_1 = m_2$)



Perpendicular lines
($m_1 = -1/m_2$)



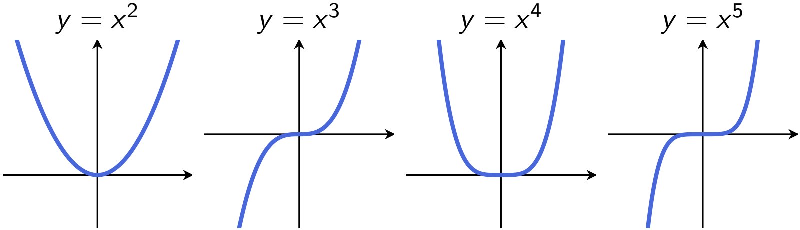
Other good functions to know: polynomials.

$$y = a_0 + a_1 x + \cdots + a_n x^n$$

(n is the *degree*)

The basics (know these graphs!)

$n = 0$:	$n = 1$:	$n = 2$:
constants	lines	parabolas



Other good functions to know: polynomials.

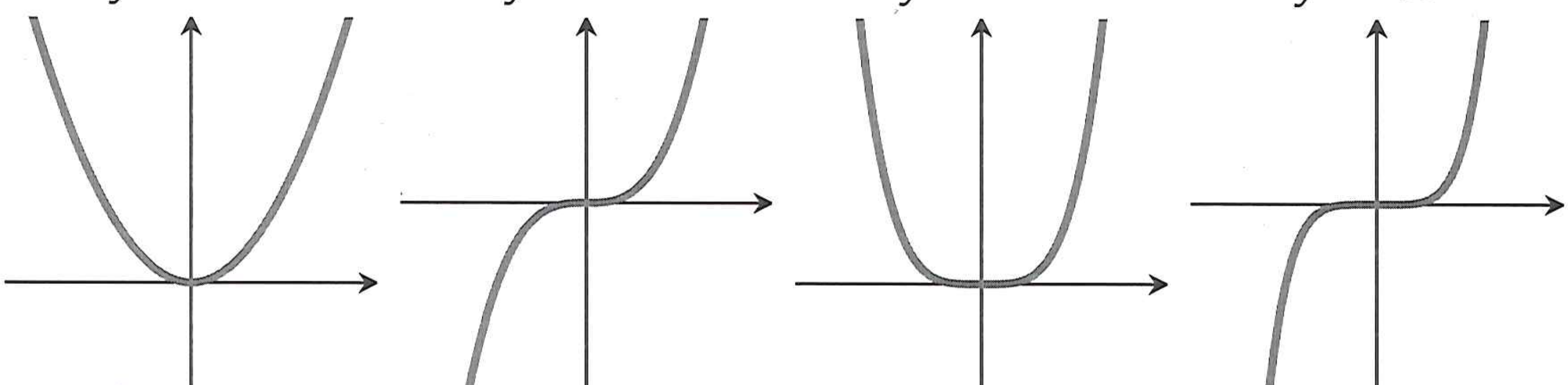
$$y = a_0 + a_1 x + \cdots + a_n x^n$$

(n is the *degree*) $a_n \neq 0$.

The basics (know these graphs!)

$n = 0$:	$n = 1$:	$n = 2$:
constants	lines	parabolas

$y = x^2$ $y = x^3$ $y = x^4$ $y = x^5$



D: $(-\infty, \infty)$
R: $[0, \infty)$

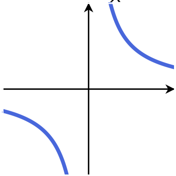
D: $(-\infty, \infty)$
R: $(-\infty, \infty)$

Other good functions to know: rationals.

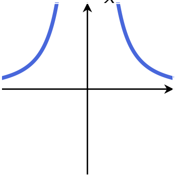
$$y = \frac{a_0 + a_1 x + \cdots + a_n x^n}{b_0 + b_1 x + \cdots + b_m x^m}$$

The basics (know these graphs!)

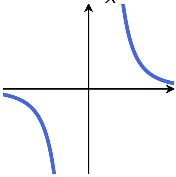
$y = \frac{1}{x}$



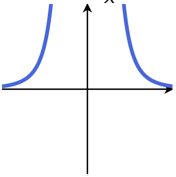
$y = \frac{1}{x^2}$



$y = \frac{1}{x^3}$



$y = \frac{1}{x^4}$



Domain : all x's that go in

Range : all y's that come out.

$y = 2x + 3$ D: all reals, $-\infty < x < \infty$

$(-\infty, \infty)$

$[a, b] \leftrightarrow a \leq x \leq b$

$(a, b] \cup [c, d) \leftrightarrow a < x \leq b$ or $c \leq x < d$

$x^{-2} = \frac{1}{x^2}$ D: $(-\infty, 0) \cup (0, \infty)$

$x^{-2} \cdot x^{2} = x^{0} = 1$ R: $(0, \infty)$

$x^{a} \cdot x^{b} = x^{a+b}$

Other good functions to know: rationals.

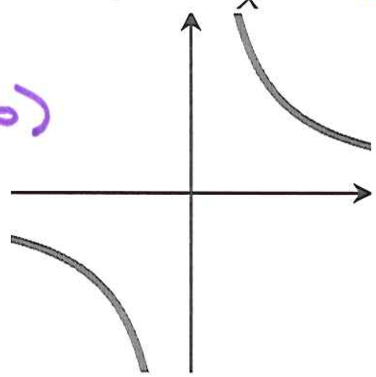
$$y = \frac{a_0 + a_1 x + \cdots + a_n x^n}{b_0 + b_1 x + \cdots + b_m x^m}$$

The basics (know these graphs!)

$y = \frac{1}{x} = x^{-1}$

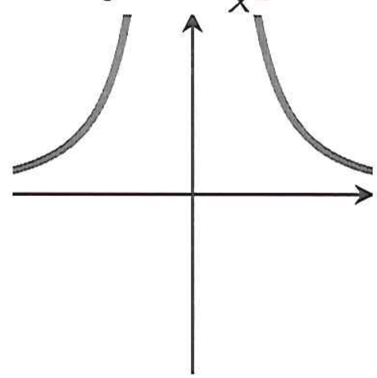
D: all $x \neq 0$
$(-\infty, 0) \cup (0, \infty)$

R: $(-\infty, 0)$
$\cup (0, \infty)$

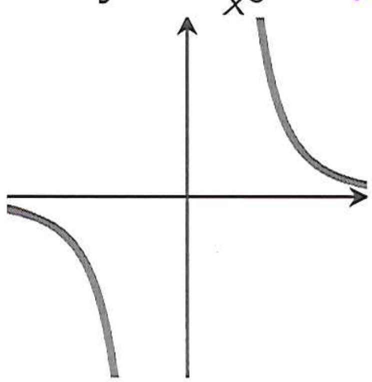


$y = \frac{1}{x^2} = x^{-2}$

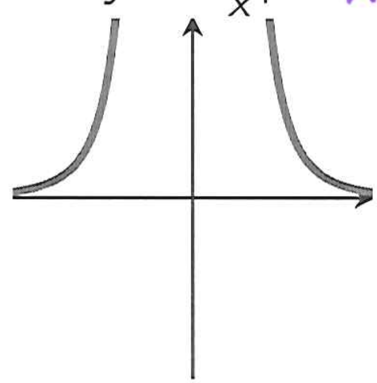
R: $(0, \infty)$



$y = \frac{1}{x^3} = x^{-3}$



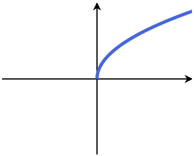
$y = \frac{1}{x^4} = x^{-4}$



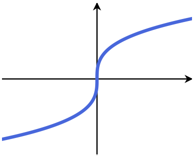
Other powers: $y = x^a$.

The basics (know these graphs!)

$y = x^{1/2} = \sqrt{x}$



$y = x^{1/3} = \sqrt[3]{x}$



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The basics (know these graphs!)

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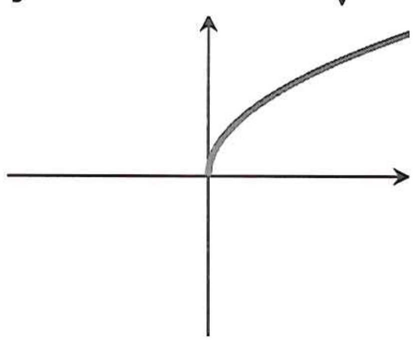

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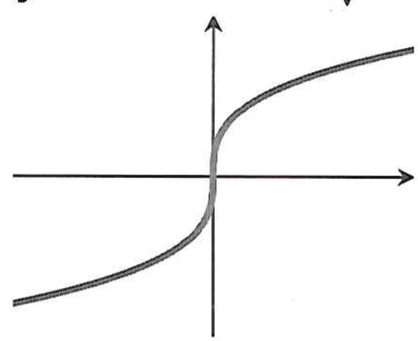
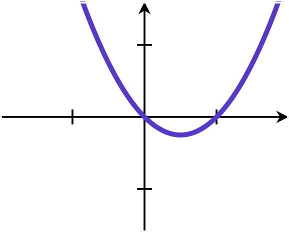


D: $[0, \infty)$

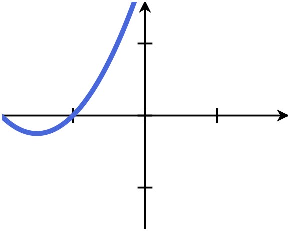
R: $[0, \infty)$ ← choice

New functions from old

Graph of $y = f(x)$:

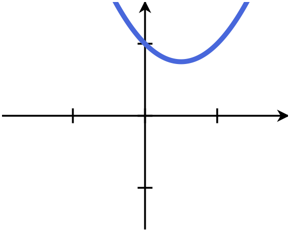


Graph of $y = f(x+2)$:



(left shift)

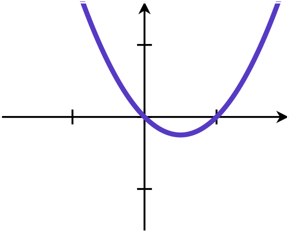
Graph of $y = f(x) + 1$:



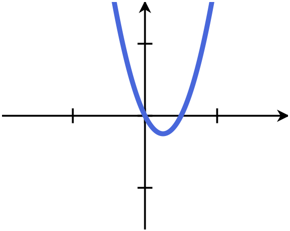
(up shift)

New functions from old

Graph of $y = f(x)$:

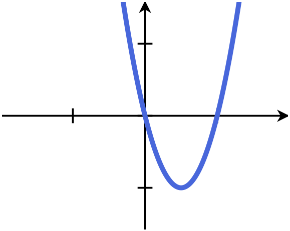


Graph of $y = f(2x)$:



(horizontal squeeze)

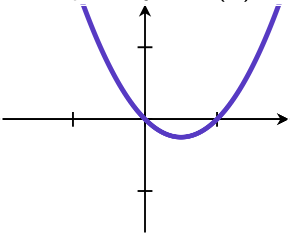
Graph of $y = 4 * f(x)$:



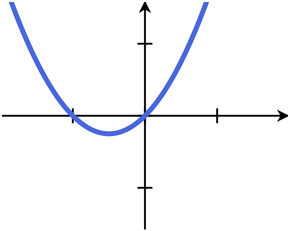
(vertical dialation)

New functions from old

Graph of $y = f(x)$:

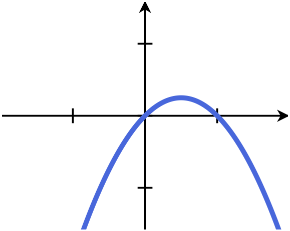


Graph of $y = f(-x)$:



(vertical reflection)

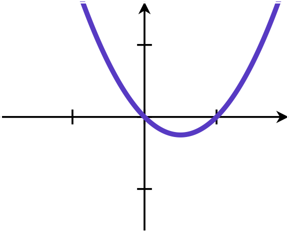
Graph of $y = -f(x)$:



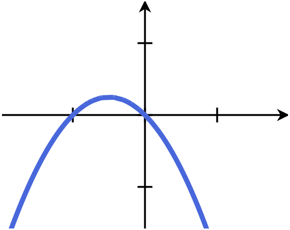
(horizontal reflection)

New functions from old

Graph of $y = f(x)$:

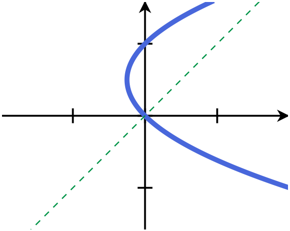


Graph of $-y = f(-x)$:



(rotation 180°)

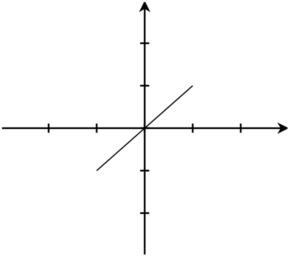
Graph of $x = f(y)$:



(flip over $y = x$)

Example: See notes

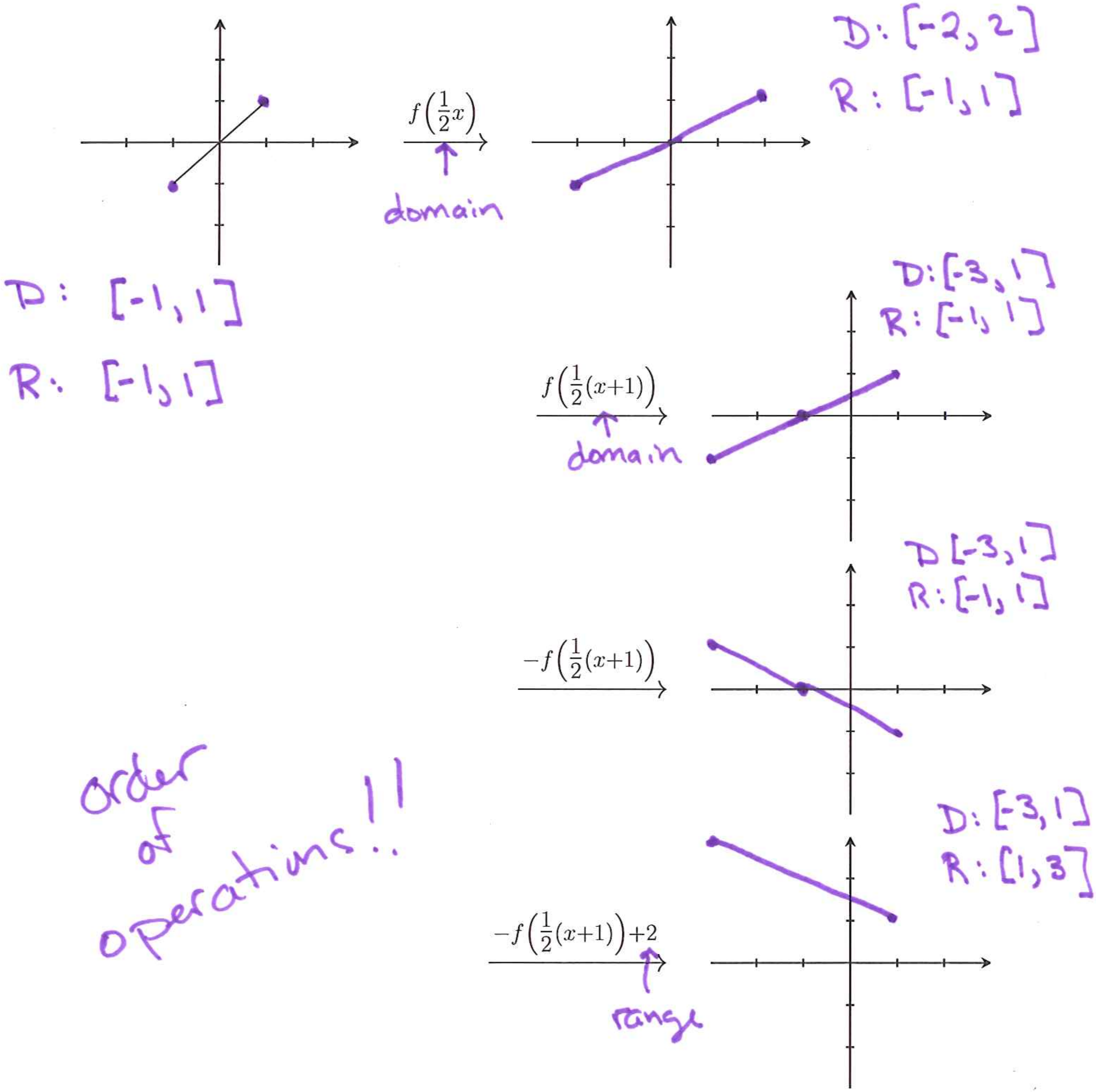
Transform the graph of $f(x)$ into the graph of $-f\left(\frac{1}{2}(x+1)\right)+2$:



$\xrightarrow{f\left(\frac{1}{2}x\right)}$ $\cdots$

The *domain* of a function f is the set of x over which $f(x)$ is defined.
The *range* of a function f is the set of y which satisfy
$y = f(x)$ for some x.

Ex: Transform the graph of $f(x)$ into the graph of $-f\left(\frac{1}{2}(x+1)\right)+2$:

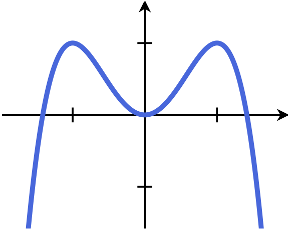


The *domain* of a function f is the set of x over which $f(x)$ is defined.

The *range* of a function f is the set of y which satisfy $y = f(x)$ for some x.

Symmetries

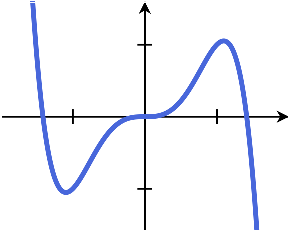
ex: $f(x) = 2x^2 - x^4$



A function $f(x)$ is *even* if it satisfies

$$f(-x) = f(x)$$

ex: $f(x) = 2x^3 - x^5$

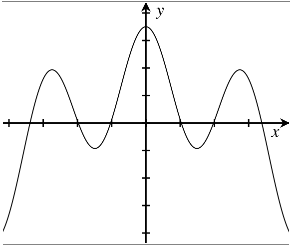


A function $f(x)$ is *odd* if it satisfies

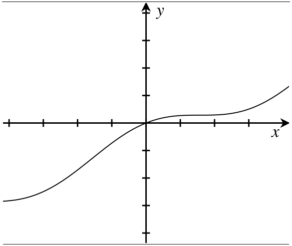
$$f(-x) = -f(x)$$

Examples: Even, odd, or neither?

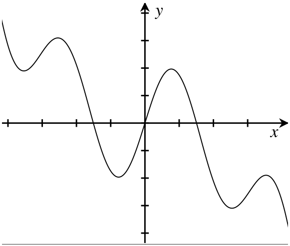
(a)



(b)



(c)



(d) $f(x) = \dfrac{x^3 + x}{x + \frac{1}{x}}$

(for this one:
actually plug in $-x$
and see what happens
algebraically)

(d) $f(-x) = \dfrac{(-x)^3 + (-x)}{(-x) + \frac{1}{(-x)}} = \dfrac{-x^3 - x}{-x - \frac{1}{x}}$

$$= \frac{-(x^3 + x)}{-(x + \frac{1}{x})}$$

$\dfrac{\text{odd}}{\text{odd}} = \text{even}$

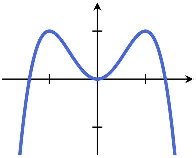
$$= \frac{x^3 + x}{x + \frac{1}{x}} = f(x)$$

even

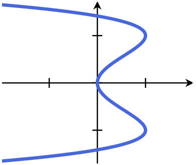
(a) even (b) neither (c) odd

A graph is a graph of a *function* if for every x in its domain, there is exactly one y on the graph which is mapped to by that x:

Function:

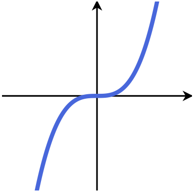


Not a function:



A function is additionally *one-to-one* if for every y, there is at most one x which maps to that y.

A one-to-one function:



Inverse functions

Let f be a one-to-one function.
If g is a function satisfying

$$f(g(x)) = g(f(x)) = x$$

then g is the *inverse function* of f. Write $g(x) = f^{-1}(x)$.

To calculate $f^{-1}(x)$, set $f(y) = x$ and solve for y. Then $y = f^{-1}(x)$.

To get the graph of $f^{-1}(x)$, flip the graph of $f(x)$ over the line $y = x$.

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Example: If $f(x) = x^3$, then $f^{-1}(x) = \sqrt[3]{x}$

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Example: If $f(x) = \frac{2x}{x-1}$,

$$\text{solve } x = \frac{2y}{y-1} \text{ for } y \text{ to get } y = \frac{x}{x-2}.$$

So $f^{-1}(x) = \frac{x}{x-2}$.

To get the graph of $f^{-1}(x)$, flip the graph of $f(x)$ over the line $y = x$.

Pair up graphs with their inverses: