Hour Exam #2

Math 3

November 6, 2013

Name (Print): Solutions
Last First

On this, the second of the two Math 3 hour-long exams in Fall 2013, and on the final exam, I will work individually, neither giving nor receiving help, guided by the Dartmouth Academic Honor Principle.

Signature:

Section (circle):

10 (Daugherty) 11 (Daugherty) 12 (Diesel) 2 (Diesel)

Instructions: You are not allowed to use calculators, books, or notes of any kind. All of your answers must be marked on the Scantron form provided or entered on the test, depending on the problem. Take a moment now to print your name and section clearly on your Scantron form and on page 1 of your exam booklet and sign the affirmation. You may write on the exam, but you will only receive credit on Scantron (multiple-choice) problems for what you write on the Scantron form. At the end of the exam, you must turn in both your Scantron form and your exam booklet. There are 13 multiple-choice problems worth 4 points each and 3 long-answer written problems worth a total of 48 points.

Problem	Points	Score
~~16~~ 14	~~15~~ 16	
~~17~~ 15	~~10~~ 12	
~~18~~ 16	~~18~~ 20	
Tot. LA	48	
Tot. MC	52	
Total	100	

1. Let $f(x) = x^3 - 3x$. Consider the following statements.

   ~~(I)~~ $f(x)$ has no inflection points.

   (II) $f(x)$ is increasing on the intervals $(-\infty, -1]$ and $[1, \infty)$.

   ~~(III)~~ $f(x)$ has a local maximum at $x = 1$.

   Which of the above statements are correct?

   (a) I only

   (b) II only

   (c) III only

   (d) II and III only

   (e) all statements are true

2. What is the maximum value of $f(x) = x^3 - 3x$ (the same function as in the last problem) over the interval $[-1, 3]$?

   (a) 18

   (b) 3

   (c) 2

   (d) -2

   (e) None of the above.

3. Find the linearization of $y = \sqrt{x}$ at the point $(25, 5)$ and use it to estimate $\sqrt{26}$.

(a) $\sqrt{26} \approx 5 + \frac{1}{25}$

(b) $\sqrt{26} \approx 5 + \frac{1}{50}$

(c) $\sqrt{26} \approx 5 - \frac{1}{5}$

(d) $\sqrt{26} \approx 5 - \frac{1}{10}$

(e) None of the above.

4. Use properties of integrals to determine which of the following is equal to the integral

$$\int_{-5}^{5} (x-1)^3 + \sin(x)\,dx.$$

(a) $\displaystyle\int_{-5}^{5} \sin(x)\,dx$

(b) $\displaystyle\int_{-5}^{5} (x-1)^3\,dx$

(c) $\cos(x) + C$

(d) $\displaystyle 2\int_{0}^{5} (x-1)^3\,dx$

(e) None of the above.

5. Which of the following is equal to $\displaystyle\int_{-1}^{2} x^3\,dx$?

(a) $\displaystyle\lim_{n\to\infty} \sum_{i=0}^{n-1} (-1 + \tfrac{3i}{n})^3 (\tfrac{3}{n})$

(b) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} (\tfrac{3i}{n})^3 (\tfrac{3}{n})$

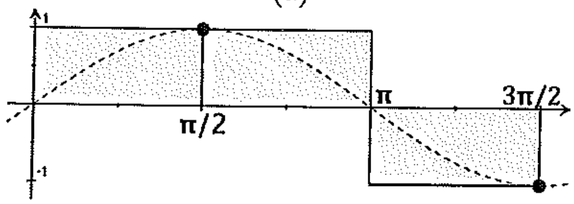
(c) $\displaystyle\lim_{n\to\infty} \sum_{i=0}^{n-1} (-1 + (\tfrac{3i}{n})^3)(\tfrac{3}{n})$

(d) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{4}(-1 + \tfrac{3i}{n})^4 (\tfrac{3}{n})$

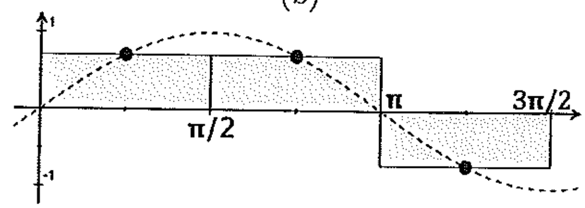
(e) None of the above.

6. Which of the following depicts an approximation of $\int_0^{3\pi/2} \sin(x)\,dx$ using Simpson's rule?

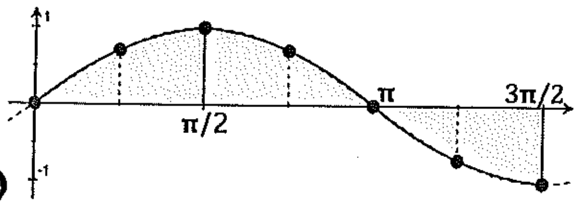
(a)

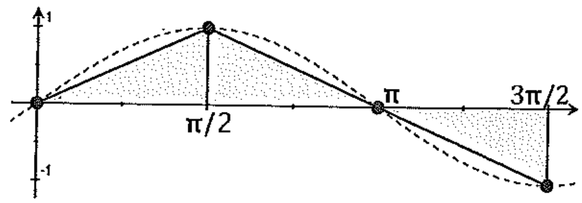


(b)



(c)





(e) None of the above.

7. Perform one iteration using Newton's Method with $x_0 = 2$ to approximate a root of $f(x) = e^{\frac{1}{2}x} - 4$. Give your value for x_1 (be careful to simplify).

(a) $\dfrac{\frac{3}{2}e - 8}{e-4}$

(b) $2\ln 4$

(c) $\dfrac{1}{2}e$

(d) $\dfrac{8}{e}$

(e) None of the above.

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

(from $0 = f'(x_0)(x_1 - x_0) + f(x_0)$

$f(2) = e - 4$

$f'(2) = \frac{1}{2}e^{1/2 x}\Big|_{x=2} = \frac{1}{2}e$

$$x_1 = 2 - \frac{e-4}{\frac{1}{2}e} = 2 - \left(2 - \frac{8}{e}\right) = 8/e$$

8. $\int 2x^{-1} + 5x^{1/2} dx =$

(a) $-2x^{-2} + \frac{5}{2}x^{-1/2} + C$

(b) $-2x^{-2} + \frac{5}{2}x^{3/2} + C$

(c) $2\ln|x| + \frac{10}{3}x^{3/2} + C$

(d) $2\ln|x| + \frac{15}{2}x^{3/2} + C$

(e) None of the above.

9. $\int \frac{(\ln x)^5}{x} dx =$

(a) $\frac{(\ln x)^6}{6x} + C$

(b) $\frac{u^6}{6} + C$

(c) $\frac{1}{6}(\ln x)^6 + C$

(d) $\frac{(\ln x)^6}{3x^3} + C$

(e) None of the above.

10. $\displaystyle\int \frac{1+\sin(x)}{\cos^2(x)}dx =$

(a) $\tan(x) + \sec(x) + C$

(b) $\sec(x) + C$

(c) $\tan + \frac{1}{3\sin^3} + C$

(d) $\frac{x+\sin(x)}{\cos^2(x)} + C$

(e) None of the above.

11. $\displaystyle\frac{d}{dx}\int_3^{5x} \tan^{-1}(t)\,dt =$

(a) $5\tan^{-1}(5x)$

(b) $\tan^{-1}(5x) - \tan^{-1}(3)$

(c) $\dfrac{\tan^{-1}(5t)}{5}$

(d) $\dfrac{1}{1+25x^2} - \dfrac{1}{10}$

(e) None of the above.

12. A spherical balloon is inflated at a rate of 200 cubic centimeters per second. How fast is the radius changing when the radius equals 3 centimeters?

(a) $\frac{36}{\pi}$ centimeters per second

(b) $\frac{4}{9\pi}$ centimeters per second

(c) $\frac{3\pi}{50}$ centimeters per second

(d) $\frac{50}{9\pi}$ centimeters per second

(e) None of the above.

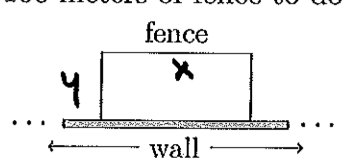
Know $\frac{dV}{dt} = 200$

want $\left.\frac{dr}{dt}\right|_{r=3}$

$V = \frac{4}{3}\pi r^3 \rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

So $\left.\frac{dr}{dt}\right|_{r=3} = 200/4\pi(3)^2 = \boxed{50/9\pi}$

13. Suppose you want to fence in a rectangular area against a long pre-existing wall, and you have 100 meters of fence to do it. What the maximum area you can enclose?



(a) 25 square meters

(b) 50 square meters

(c) $25 \cdot 25$ square meters

(d) $25 \cdot 50$ square meters

(e) None of the above.

Know $100 = 2y + x$

$\hookrightarrow x = 100 - 2y$

$= 2(50-y)$

maximize

$A = xy$

$= 2(50-y)y$

$= 2(50y - y^2)$

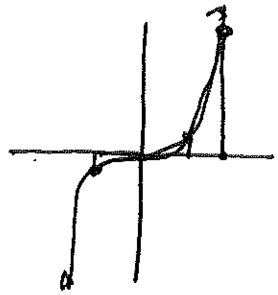
So $\frac{dA}{dy} = 2(50-2y) = 4(25-y)$

$0 \leq y \leq 50$

y	A
0	0
50	0
25	$\boxed{2 \cdot 25 \cdot 25}$
	max

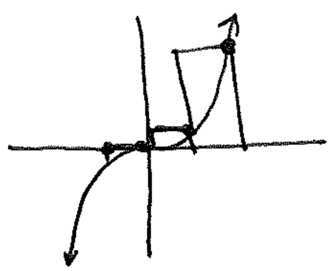
Long answer questions

14. (a) Estimate the definite integral $\int_{-1}^{2} x^3\,dx$ using three trapezoids of equal width.



$$A = 1 * \tfrac{1}{2}\left((-1)^3 + 2\cdot 0^3 + 2\cdot 1^3 + 2^3\right)$$
$$= \tfrac{1}{2}(-1 + 2 + 8) = \boxed{9/2}$$

(b) Give an over-estimate of $\int_{-1}^{2} x^3\,dx$ using three rectangles of equal width.



$$A = 1 * (0 + 1 + 8) = \boxed{9}$$

(c) Use the fundamental theorem of calculus to calculate $\int_{-1}^{2} x^3\,dx$.

$$\int_{-1}^{2} x^3\,dx = \tfrac{1}{4}x^4\Big|_{x=-1}^{2} = \frac{2^4}{4} - \frac{(-1)^4}{4} = 4 - 1/4 = \frac{15}{4}$$

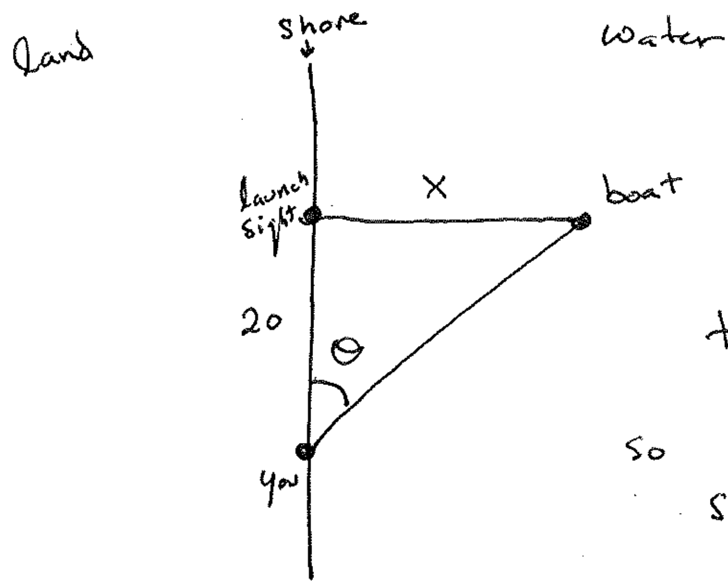
(d) Give an interval $[a, b]$ different from $[-1, 2]$ such that $\int_{-1}^{2} x^3\,dx = \int_{a}^{b} x^3\,dx$.

Since $\int_{-1}^{1} x^3\,dx = 0$

$$\int_{-1}^{2} x^3\,dx = \int_{-1}^{1} x^3\,dx + \int_{1}^{2} x^3\,dx = \int_{1}^{2} x^3\,dx$$

$$\boxed{[1, 2]}$$

15. You are standing on a straight shoreline, 20 meters away from the launch sight of a boat, also on the shoreline. The boat starts sailing straight out to sea (perpendicular to the shoreline) at a speed of 5 meters per second. How fast is the angle between your line of sight to the boat and the shoreline changing when the boat is 10 meters away from the shore?



Know: $\frac{dx}{dt} = 5$

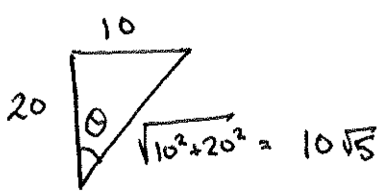
Want: $\left.\frac{d\theta}{dt}\right|_{x=10}$

$$\tan\theta = \frac{x}{20}$$

So

$$\sec^2\theta \, \frac{d\theta}{dt} = \frac{1}{20}\frac{dx}{dt}$$

So $$\left.\frac{d\theta}{dt}\right|_{x=10} = \frac{1}{20}\cdot\left.\frac{dx}{dt}\right|_{x=10}\cdot\left(\sec\theta\big|_{x=10}\right)^{-2}$$



$$= \frac{1}{20}\cdot 5\cdot\left(\frac{\sqrt{5}}{2}\right)^{-2}$$

$$\sec\theta\big|_{x=10} = \frac{10\sqrt{5}}{20} = \frac{\sqrt{5}}{2}$$

$$= \frac{4\cdot 5}{20\cdot 5} = \boxed{\frac{1}{5}}$$

16. For this problem, show all work and justify your steps in order to receive full credit.

Let $f(x) = \dfrac{1}{x^2-1}$. $= \dfrac{1}{(x+1)(x-1)}$

(a) What are the horizontal and vertical asymptotes of $f(x)$? If there are none, write 'none'.

vertical : $\boxed{x = \pm 1}$

$$\left(\lim_{x\to 1^{\pm}} \frac{1}{(x+1)(x-1)} = \pm\infty \qquad \lim_{x\to -1^{\pm}} \frac{1}{(x+1)(x-1)} = \mp\infty \right)$$

horizontal: $\lim_{x\to\pm\infty} \dfrac{1}{x^2-1} = \boxed{1 = y}$

(b) What are the intervals of increase and decrease for $f(x)$?

$$f'(x) = \frac{-2x}{(x^2-1)^2}$$

+	u	+	0	−	u	−
	−1		0		1	
↗		↗		↘		↘

incr : $(-\infty, -1) \cup (-1, 0]$

decr : $[0, 1) \cup (1, \infty)$.

(c) What are the intervals on which $f(x)$ is concave up? Concave down?

$$f'' = \frac{-2(x^2-1)^{\cancel{2}} + 2x\cdot 2\cancel{(x^2-1)}\cdot 2x}{(x^2-1)^{\cancel{4}\,3}} = -2\,\frac{x^2-1 \quad 4x^2}{(x^2-1)^3}$$

$$= -2\,\frac{-3x^2-1}{(x^2-1)^3} = 2\left(\frac{3x^2+1}{(x^2-1)^3}\right)$$

+	u	−	u	+
	−1		1	

cc up : $(-\infty, -1) \cup (1, \infty)$

cc down : $(-1, 1)$

(d) Does $f(x)$ have local maximums? Minimums? If so, list them and the function value at these points. If not, write 'none'.

local max @ $x=0$, $y = \frac{1}{-1} = -1$

no local mins

(see pt (b))

(e) Using <u>all</u> the previous information, sketch the graph of $f(x)$:

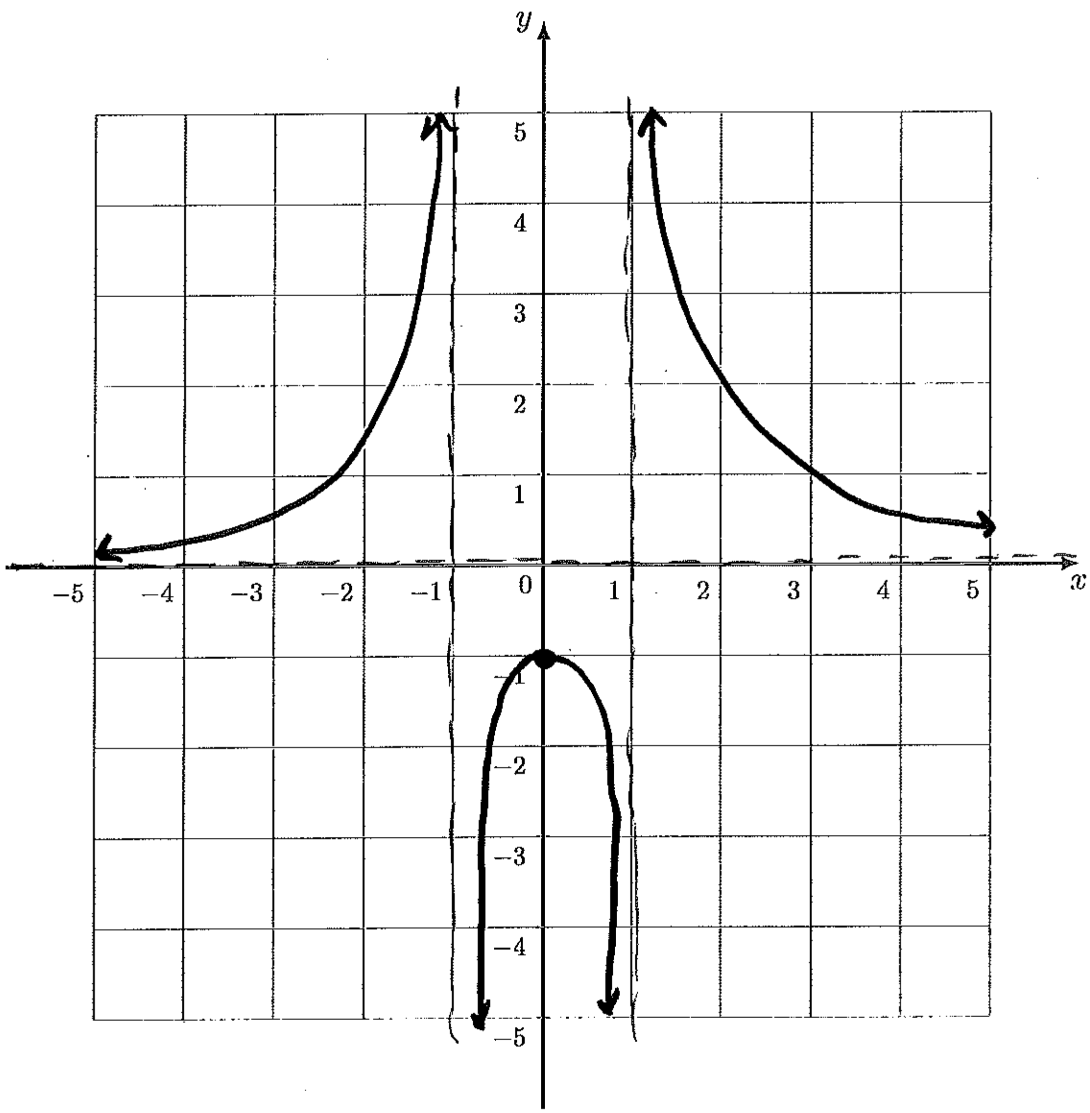
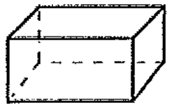
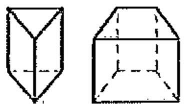
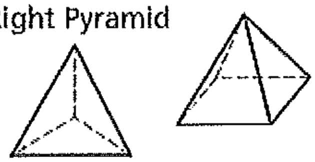
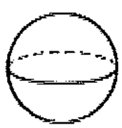


Figure	Formulas for Volume (V) and Surface Area (SA)
Rectangular Prism 	$V = lwh$ = length × width × height $SA = 2lw + 2hw + 2lh$ = 2(length × width) + 2(height × width) + 2(length × height)
General Prisms 	$V = Bh$ = area of base × height SA = sum of the areas of the faces
Right Circular Cylinder 	$V = Bh$ = area of base × height $SA = 2B + Ch$ = (2 × area of base) + (circumference × height)
Right Pyramid 	$V = \frac{1}{3}Bh = \frac{1}{3}$ × area of base × height $SA = B + \frac{1}{2}P\ell$ = area of base + ($\frac{1}{2}$ × perimeter of base × slant height)
Right Circular Cone 	$V = \frac{1}{3}Bh = \frac{1}{3}$ × area of base × height $SA = B + \frac{1}{2}C\ell$ = area of base + ($\frac{1}{2}$ × circumference × slant height)
Sphere 	$V = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \pi \times$ cube of radius $SA = 4\pi r^2 = 4 \times \pi \times$ square of radius