Written Assignment # 16

Math 38

Due: Wednesday, May 11, 2005

In preparation for Exam 2. Review: 4.1 and 4.2

1. Give equations that relate the value $\kappa(G)$ and the values $\kappa'(u, v)$ where $u, v \in V(G)$.

2. If v is a vertex incident with a cut-edge in a connected graph G, then find necessary and sufficient conditions so that this vertex is a cut-vertex.

3. If T is a tree what vertices are cut-vertices? Which tree has the maximum number of cut-vertices and which one has the minimum number of cut-vertices?

4. Is it true that every graph of order 3 or more that contains a cut-edge also contains a cut-vertex?

5. What can you say about a vertex x that is on every path from a vertex u and a vertex v in a connected graph G?

6. Is it true that every nontrivial connected graph contains at least two vertices that are not cut-vertices?

7. Give an example of a simple connected graph that contains more cut-edges than cut-vertices and one that contains more cut-vertices than cut-edges.

8. Show that if G is not k connected, then G contains a set of vertices U such that $G - U$ is disconnected and $|U| = k - 1$.

9. We proved in class that: G is 2-connected if and only if for every pair of vertices $u, v \in V(G)$ then u and v lie in a common cycle of G. Does this generalize for k-connected graphs, where $k \geq 2$?

10. In class we showed that using vertex form of Menger's theorem there is a characterization of k-connected graphs in terms of internally disjoint paths. Can you give a similar characterization for k-edge-connected graphs?

11. What is the edge-connectivity of the n-cube?