

Math 36

Homework 03

Differential Equations: Predator-Prey Ecosystem Model

1. Describe the competitive hunters model (handout): describe the assumptions in your own words, explain the mathematical model, analyze the slope graph, and interpret the long-term behavior of the model.
2. Consider an ecosystem consisting of clover, rabbits, foxes, and lions. Create a system of differential equations that models their populations, taking into consideration the following assumptions:

Clover. The number of clover plants (C) fluctuates periodically according to time. In the winter months, there are approximately 5 thousand clover plants, and in the summer, 105 thousand. In addition, since rabbits eat clover, the rate of change is negatively proportional to the number of rabbits.

Rabbits. When the number of rabbits (R) is small, the rate of increase in the population is proportional to the current population. The carrying capacity of the environment for rabbits is one rabbit for every 200 clover plants. The rate of decrease in the population is proportional to the rabbit-fox interaction as well as the rabbit-lion interaction.

Foxes. Let F denote the number of foxes. When the ratio of foxes to rabbits is small (less than k), the rate of increase in the fox population is proportional to the fox population; near k , there is little change; greater than k , the population decreases (due to overcompetition). In addition, the population decreases in proportion to fox-lion interaction.

Lions. Let L denote the number of lions. Lions eat rabbits and foxes (but not clover). To a lion, 1 fox is equivalent to 3 rabbits. When the ratio of lions to their total prey is small (less than ℓ), the rate of increase in the lion population is proportional to the lion population; near ℓ , there is little change; greater than ℓ , the population decreases (due to overcompetition).