Math 35

Winter 2014

Some Proof Principles

Generally, proving something requires some creativity; there is no recipe for producing a proof. However, there are some standard techniques that can be used, depending on the form of the statement you are trying to prove. (Note that "can" does not mean "must.") Here are a few of them.

1. To prove a statement of the form "if A, then B," assume A and prove B. Or, prove the *contrapositive*, "if not B, then not A," by assuming not B and proving not A.
2. To prove a statement of the form "not A," use *proof by contradiction*: Assume A, and deduce a contradiction, something obviously false or contradictory.
3. To prove a statement of the form "for all real numbers x, A(x)," let x be a name for an arbitrary real number, and prove A(x).
4. To prove a statement of the form "there is a real number x such that A(x)," find a specific example c and prove that A(c). (For example, prove that A(0).)
5. To prove a statement of the form "A and B," prove both A and B.
6. To prove a statement of the form "A or B," prove "If not A, then B," or prove "If not B, then A," or assume "not A and not B" and deduce a contradiction. Or, consider all possible cases, and prove that in some cases A holds, and in other cases B holds.
7. In general, prove something by considering all possible cases separately. You must be sure the cases you list cover all possibilities.
8. To prove something is unique, assume there are two such things, and prove they are actually equal.
9. To prove a statement of the form "there is a unique x such that A(x)," prove both "there is an x such that A(x)" and "the x such that A(x) is unique." This is called proving existence and uniqueness.
10. To prove a statement of the form "for every natural number n, A(n)," use proof by mathematical induction: Prove A(1); then, assume that n is a natural number such that A(n), and prove A(n+1).