Math 35
Winter 2014

Writing Proofs

For an example, look at the proof of the first "additional exercise" on the in-class exercises from Wednesday, January 8.

1. Proofs are written in mathematical English. This means you should use complete sentences with correct grammar and punctuation.

   On homework, you can be a little more informal; complete sentences are not always necessary, but complete thoughts are. For example, it's fine to write "Proof: By induction," rather than "Proof: We will prove this by induction."

2. You should use mathematical formulas, equations, and pictures in a proof, whenever they help make your proof readable and understandable.

3. Formulas, equations, and pictures should always be explained. A string of equations without explanations is not a proof.

4. Formulas and equations are included in sentences, and must be punctuated accordingly. Notice the punctuation in the example proof.

5. Always begin by stating the proposition you are going to prove.

6. Make the logic of your proof clear to your reader. If you are proving the additive identity of a field is unique, it is better to begin with, "Let a be an additive identity. We will prove that $a = 0$," than to begin with merely, "Let a be an additive identity."

7. How your proof is laid out on the paper matters. Centering equations on their own lines, and skipping lines between parts of a solution, can make your solution much more readable. Neatness counts. Don't be afraid to use extra sheets of paper when writing up your homework.

8. It is fine to use formulas and results from the text or from class (unless the assignment specifies otherwise). Be sure you reader knows what axiom, formula, or result you are using.

9. There is generally more than one correct proof of a theorem, and more than one way to write up a given proof. Unless a homework or exam problem specifies a particular approach or technique, you can use any logically valid method of proof.

10. The amount of detail needed in a proof depends on the intended reader. For this class, your intended reader should be a student in the class who does not understand the material quite as well as you do.

11. The mathematical "we" is common in proofs, but it is fine to use "I," as in, "Let a be an additive identity. I will prove that $a = 0$."

12. Professor Annalisa Crannell of Franklin and Marshall College has written a booklet about writing mathematics for her calculus classes. She discusses a number of strategies and conventions for writing mathematics well. You can find her booklet here:

    https://edisk.fandm.edu/annalisa.crannell/writing_in_math/guide.pdf

    Professor Steven Kleiman of MIT has written a more advanced guide to writing mathematics, intended for undergraduate students who are writing mathematical papers. You can find his guide here:

    http://www.mit.edu/afs/athena.mit.edu/course/other/mathp2/www/piil.html

13. Excellent mathematical writing style embodies several characteristics, of which the three most important are clarity, clarity, and clarity. It is important to use words precisely and correctly. Generally, simple declarative sentences and consistent word use are preferable to variation in sentence structure and vocabulary. The same is true of most technical writing; the deeper and more complex the ideas, the more simple the writing should be. My favorite quotation about this comes from the web page "Guidelines for Writing a Phiilosophy Paper" by NYU philosophy professor James Pryor:[1]

    > If your paper sounds as if it were written for a third-grade audience, then you've probably achieved the right sort of clarity.

---

[1]http://www.jimpryor.net/teaching/guidelines/writing.html