Math 35
Winter 2014
Homework Assigned Monday, January 6

As usual, this homework is due at the beginning of class next Monday, January 13.

Here are some propositions about inequalities that can be proven from the axioms for an ordered field. Your homework is to provide proofs for Propositions 2 and 5. You may use basic facts about arithmetic (that is, addition, multiplication, subtraction, division, etc.) that do not have to do with inequalities.

Be sure to begin each proof by giving a statement of the proposition you are proving.

Note that $x > y$ is just another way of writing $y < x$.

Note also that the end of a proof is designated by the symbol $\square$.

Proposition 1: If $a < b$, then $a + c < b + c$.

Proof: Suppose that $a < b$. By (3) of Definition 1.3, $b - a > 0$. Since by basic arithmetic

$$b - a = (b + c) - (a + c),$$

we have $(b + c) - (a + c) > 0$. But then, by (3) of Definition 1.3 again, $a + c < b + c$. $\square$

Proposition 2: If $a > 0$ then $-a < 0$, and if $a < 0$ then $-a > 0$.

Proof: Homework.

Proposition 3: $0 < 1$.

Proof: First, suppose that $0 < -1$. Then, by (2) of Definition 1.3, $1 = (-1)(-1) > 0$. But then, by Proposition 2, since $1 > 0$ we have $-1 < 0$, contradicting our original supposition that $0 < -1$ (and (1) of Definition 1.2, which implies that $0 < -1$ and $-1 < 0$ cannot both be true).

Since the supposition that $0 < -1$ leads to a contradiction, we know $0 \not< -1$. By basic arithmetic, $0 \neq -1$. Therefore, by (1) of Definition 1.2, we must have $-1 < 0$.

Now, by Proposition 2 again, we can conclude that $1 = -(-1) > 0$. $\square$

Proposition 4: If $a < b$ and $c > 0$ then $ac < bc$.

Proof: Suppose $a < b$. Then by (3) of Definition 1.3, $0 < b - a$. Since $0 < c$, by (2) of Definition 1.3 we have $0 < c(b - a) = bc - ac$. Therefore, by (3) of Definition 1.3, $ac < bc$. $\square$

Proposition 5: If $a < b$ and $c < 0$ then $ac > bc$.

Proof: Homework. $\square$

Proposition 6: If $a \neq 0$ then $a^2 > 0$.

Proof: By (1) of Definition 1.2, either $a > 0$ or $a < 0$. If $a > 0$, then $a^2 = a \cdot a > 0$ by (2) of Definition 1.3. If $a < 0$, then by Proposition 5, multiplying by the negative number a reverses the inequality $a < 0$; that is, $a^2 = a \cdot a < a \cdot 0 = 0$. $\square$