

m33s06: Homework 4

1 (a) Use the Fourier and Laplace transforms to find the Green's function for the Diffusion Equation with decay term, i.e. the solution to the IVP

$$\begin{cases} u_t - cu_{xx} + \lambda u = 0 \\ u(x, 0) = \delta(x). \end{cases}$$

(b) $2kg$ of a radioactive compound is randomly distributed along the x -axis and then diffuses-decays according to the above model with $c = 1$, $\lambda = \ln 2$ and t measured in days. How much compound is left after 3 days?

(c) The contamination due to the compound is considered negligible at (x, t) if $u(x, t) < 0.1$. Estimate how long must pass before the contamination is negligible everywhere. (You may find it helpful here to create new procedures by copy/pasting those from the Diffusion MAPLE and adjusting the formula for $\mathbf{u} := (\mathbf{x}, \mathbf{t}) \rightarrow \dots$ to include the decay term).

Notes:

(1) A useful result is that $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$