m33s06: Homework 2

1 (a) Solve the initial value problem

$$y'' + 2y' + 2 = \sum_{n=1}^{\infty} \delta(t-n), \quad y(0) = 0, y'(0) = 0$$

by direct use of the Laplace transform. (The table on p.203 may be useful - your answer should involve a sum.) Note this represents a vibrating spring-mass, that is receiving an impulse at regular intervals.

Plot your solution for $0 \leq t \leq 10$ using MAPLE. (If you are careful here, you don't need to use the whole sum, which will make the computation much easier).

(b) Find a fundamental solution u for the operator $D^2 + 2D + 2$ which satisfies $u(0) = 0$.

(c) A spring-mass system is initially (i.e at $t = 0$) held at rest in its equilibrium position. The threshold for the vibration to be considered negligible is when its amplitude is always less than 0.01.

The following forcing function

$$f(t) = \begin{cases} 1, & t < 1 \\ 1/t^2, & t \geq 1 \end{cases}$$

is applied to the system. By graphing a solution, estimate when the vibration becomes negligible. (Using a numerical convolution may speed things up here.)

2 Use the Fourier transform (and the questions from 2.7) to find a fundamental solution for the operator $D^2 + 2D$.

(No credit will be given to a method not based on the Fourier transform.)

Notes: (1) The MAPLE syntax for sum $\sum_{k=1}^{m} f(k)$ is `sum(f(k),k=1..m)`

(2) On the most recent MAPLE worksheet for download is the `nLapconvolve` routine. The syntax is the same as for `Lapconvolve` except MAPLE will only apply numerical methods rather than trying to evaluate symbolically.