

MAJOR FACTS ABOUT PERMUTATION GROUPS

- FACT 1. (**Products of Disjoint Cycles**) Every permutation of a finite set can be written as a cycle or as a product of disjoint cycles.
- FACT 2. (**Disjoint Cycles Commute**) Let $\alpha = (a_1, a_2, \dots, a_m)$ and $\beta = (b_1, b_2, \dots, b_n)$ be two cycles. **If** α and β are disjoint (that is, have no common entries), **then** $\alpha\beta = \beta\alpha$.
- FACT 3. (**Order of a Permutation**) Let $\alpha \in S_n$ be written in disjoint cycle form. **Then** the order of α is the **least common multiple** of the lengths of the cycles.
- FACT 4. (**Product of 2-Cycles**) Every permutation in S_n for $n > 1$ is a product of (**not necessarily disjoint**) 2-cycles.
- FACT 5. (**Identity Permutation as a Product of 2-Cycles**) **If** $\varepsilon = \beta_1\beta_2 \cdots \beta_r$, where all β 's are 2-cycles, **then** r is even.
- FACT 6. (**Always Even or Always Odd**) Let $\alpha \in S_n$ such that $\alpha = \beta_1\beta_2 \cdots \beta_r$ and $\alpha = \gamma_1\gamma_2 \cdots \gamma_s$, where all β 's and γ 's are 2-cycles. **Then** $r \equiv s \pmod{2}$.
- FACT 7. (**Group of Even Permutation**) The set A_n of even permutations in S_n forms a subgroup of S_n . The order of A_n is $n!/2$.