Midterm 2 Review

Section 6.1 - Areas Between Curves

1. Find the area of the region enclosed by the following curves:
   (a) $y = e^x$, $y = x^2 - 1$, $x = -1$, $x = 1$
   (b) $y = \sin x$, $y = x$, $x = \pi/2$, $x = \pi$
   (c) $y = 1/x$, $y = 1/x^2$, $x = 2$
   (d) $x = 2y^2$, $x = 4 + y^2$
   (e) $y = \sqrt{x}$, $y = \frac{1}{2}x$, $x = 9$

Section 6.2/6.3 - Volumes

1. Find the volume of the following using the disk/washer method.
   (a) The region enclosed by $y = x^{1/3}$, $y = 0$, $x = 1$ rotated about the y-axis.
   (b) The region enclosed by $y = x^3$, $y = 8$, $x = 0$ rotated about the x-axis.
   (c) The region enclosed by $y = \sqrt{x}$, $x = 0$, $y = 2$ rotated about the x-axis.
   (d) The region enclosed by $y = x^2$, $x = y^2$ rotated about the line $y = 1$.
   (e) The region enclosed by $xy = 1$, $y = 0$, $x = 1$, $x = 2$ rotated about the line $x = -1$.
   (f) Find the volume of a sphere of radius r.
   (g) Find the volume of a cap of a sphere where the radius of the sphere is r and the height of the cap portion is h.
2. Find the volume of the following using the cylindrical shells method.
   (a) The region enclosed by $y = x^{1/3}$, $y = 0$, $x = 1$ rotated about the y-axis.
   (b) The region enclosed by $y = \sqrt{x}$, $x = 0$, $y = 2$ rotated about the x-axis.
   (c) The region enclosed by $xy = 1$, $x = 0$, $y = 1$, $y = 3$ rotated about the x-axis.
   (d) The region enclosed by $y = x^3$, $y = 0$, $x = 1$, $x = 2$ rotated about the y-axis.
   (e) The region enclosed by e^{-x^2}, $y = 0$, $x = 0$, $x = 1$ rotated about the y-axis.
   (f) Find the volume of a sphere of radues r.
3. First decide what method you would use to find the volume of the following and then do it.
   (a) The region enclosed by $y = 4x - x^2$, $y = x$ rotated about the y-axis.
   (b) The region enclosed by $y = \ln x$, $y = 1$, $y = 2$, $x = 0$ rotated about the y-axis.
   (c) The region enclosed by $y + x^2 = 1$, $y = 0$ rotated about the x-axis.
   (d) The region enclosed by $y = x^2$, $y = 6x - 2x^2$ rotated about the y-axis.
   (e) The region enclosed by $y = \sqrt{25 - x^2}$, $y = 0$, $x = 2$, $x = 4$ rotated about the x-axis.
   (f) The region enclosed by $x = 4y^2 - y^3$, $x = 0$, rotated about the x-axis.

6.5 - Average Value of a Function

1. Find the average value of the function on the given interval:

   (a) $f(x) = \sec^2(x/2),\ [0, \pi/2]$

   (b) $f(x) = \sin x - \sin 2x,\ [0, \pi]$

   (c) $f(x) = x \cdot \sin x,\ [0, \pi/2]$

   (d) $g(x) = \tan^3 x \sec x,\ [0, \pi/4]$

7.1 - Integration by Parts

1. Use Integration by Parts to evaluate the following integrals:

   (a) $\displaystyle\int t \cdot e^{-3t}\, dt$

   (b) $\displaystyle\int t^2 \cdot \sin 3t\, dt$

   (c) $\displaystyle\int e^x \cdot \cos x\, dx$

   (d) $\displaystyle\int \frac{\ln y}{\sqrt{y}}\, dy$

   (e) $\displaystyle\int \sin^{-1} x\, dx$

7.2 - Trigonometric Integrals

1. Use trig integral techniques to evaluate the following:

   (a) $\displaystyle\int \sin^2 x \cos^3 x\, dx$

   (b) $\displaystyle\int_0^{\pi/2} \sin^7 \theta \cos^5 \theta\, d\theta$

   (c) $\displaystyle\int \sin^2(\pi x) \cos^5(\pi x)\, dx$

   (d) $\displaystyle\int \tan 5x \sec^3 5x\, dx$

   (e) $\displaystyle\int_0^{\pi/4} \tan^4 t\, dt$

   (f) $\displaystyle\int_0^{\pi/4} \sec^4 \theta \tan^4 \theta\, d\theta$

   (g) $\displaystyle\int_{\pi/4}^{\pi/2} \cot^5 \theta \csc^3 \theta\, d\theta$

7.3 - Trigonometric Substitution

1. Use Trig sub to compute the following integrals:

   (a) $\displaystyle\int \frac{dx}{x^2\sqrt{5 - x^2}}$

   (b) $\displaystyle\int_0^3 \frac{x}{\sqrt{36 - x^2}}\, dx$

   (c) $\displaystyle\int_0^1 x^3 \cdot \sqrt{4 - x^2}\, dx$

   (d) $\displaystyle\int \frac{dt}{t^2\sqrt{t^2 - 16}}$

   (e) $\displaystyle\int \frac{t^5}{\sqrt{t^2 + 2}}\, dt$

Integrals

1. Decide what integration method you should use to evaluate the following integrals, then do it.

(a) $\displaystyle\int \cos x(1+\sin^2 x)\,dx$

(b) $\displaystyle\int \frac{x^3}{\sqrt{1+x^2}}\,dx$

(c) $\displaystyle\int_1^3 r^4 \cdot \ln r\,dr$

(d) $\displaystyle\int \sin^5 t \cdot \cos^4 t\,dt$

(e) $\displaystyle\int \frac{e^{\sqrt{t}}}{\sqrt{t}}\,dt$

(f) $\displaystyle\int \tan^3\theta \sec^2\theta\,d\theta$

(g) $\displaystyle\int_0^{\pi/2} \frac{\sin^2\theta \cdot \cot\theta}{\sec\theta}\,d\theta$

(h) $\displaystyle\int \cos x \cdot \cos^3(\sin x)\,dx$

(i) $\displaystyle\int \frac{x^2}{\sqrt{1-x^2}}\,dx$ (NOT on midterm)

(j) $\displaystyle\int x^3 \cdot e^x\,dx$

(k) $\displaystyle\int e^x \cdot \sin x\,dx$

(l) $\displaystyle\int \frac{\ln x}{x\sqrt{1+(\ln x)^2}}\,dx$