Wed, Jan 9, 2013

Announcements:

- WeBWork 1 is posted (Notebook, showing work)
- Tutor Sessions (in Wilder 115)
  - Sun 4-7 (Jacob)
  - Tues 7-10 (Steve)
  - Thurs 7-10 (Xander)
  - Make effort to come at the beginning
- Wed. O.H. cancelled

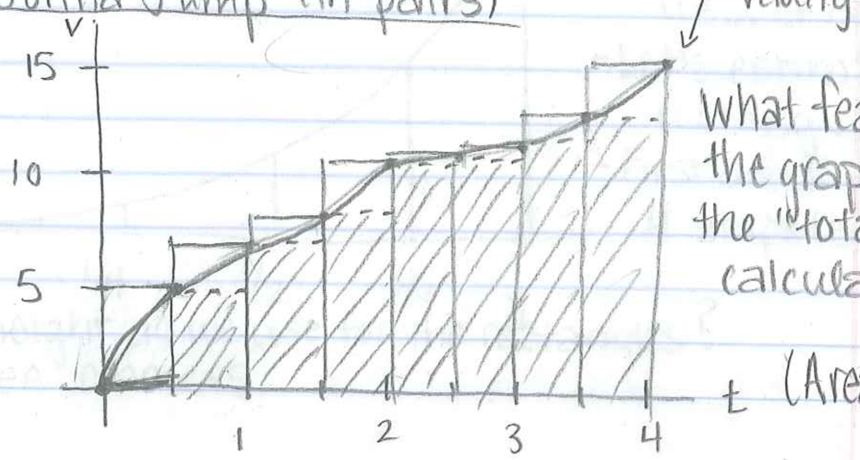
Last time: Antiderivatives

- just "going for it" isn't going to do it for us
- let's go back to what I initially posed to you: given velocity, how can we gain information on distance and position

$$\text{Position} \underset{?}{\overset{\text{can do}}{\rightleftarrows}} \text{Velocity}$$

- Great Gorilla Jump (In pairs)

- Graph it

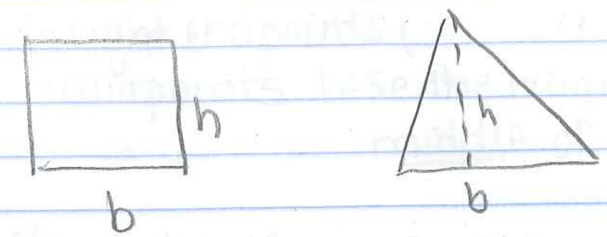


What feature of the graph is the "total distance" calculating?

(Area)

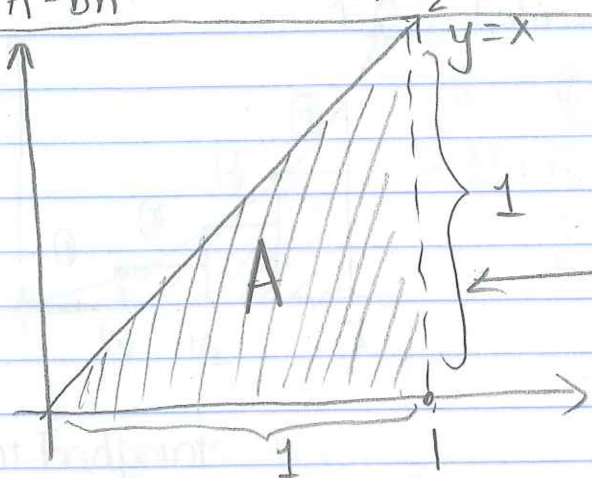
Area Under a Curve

We intuitively know what area is..., and a lot of times we already know how to calculate it.



$A = bh$ $\qquad$ $A = \frac{1}{2}bh$

Ex: 1

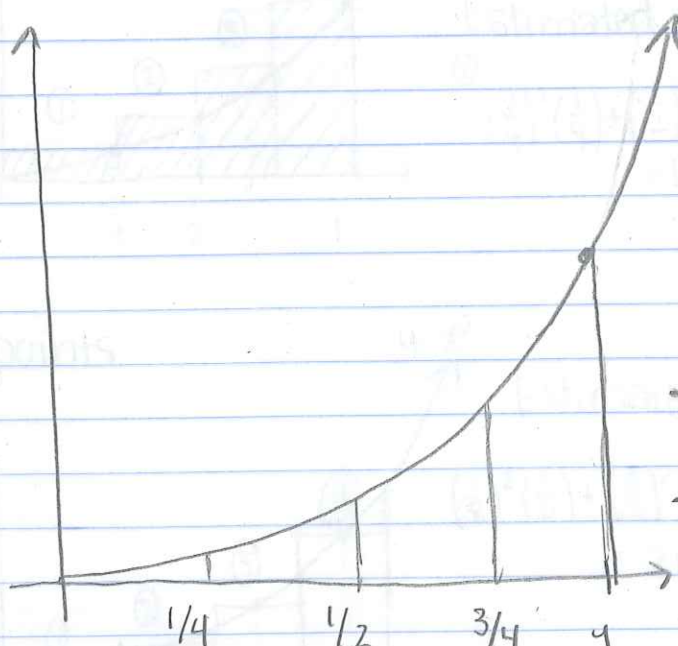


$A = \frac{1}{2}(1)(1) = \frac{1}{2}$

Area under $y=x$ between $x=0$ and 1

by Area under we mean between curve and x-axis.

2



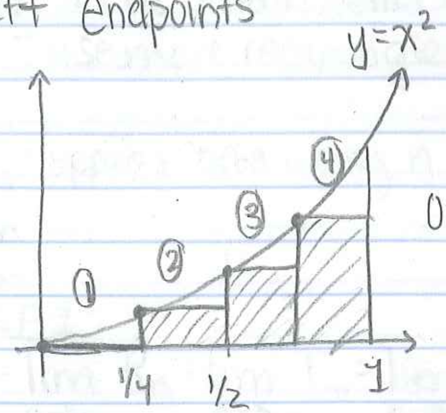
Area under $y=x^2$ between $x=0$ and $x=1$

Basic geometry fails us... or does it?
- Let's estimate using rectangles
- First divide into 4 strips (equal)

- What height do we use for the rectangles?
Three choices:

- left endpoints (use the height on the left side of the rect)
- right endpoints (" right side ")
- midpoints (use the height in the precise <u>middle</u> of the rectangle)

Left Endpoints



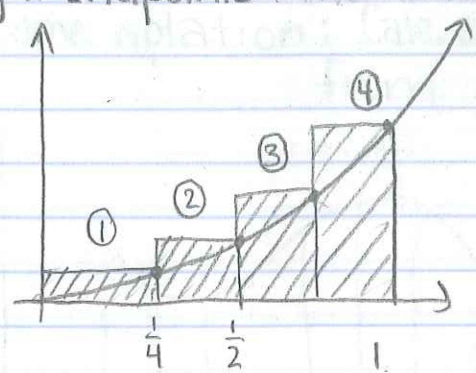
Estimated area:

$$\underset{①}{0\cdot\left(\tfrac{1}{4}\right)} + \underset{②}{\left(\tfrac{1}{4}\right)^2\left(\tfrac{1}{4}\right)} + \underset{③}{\left(\tfrac{1}{2}\right)^2\left(\tfrac{1}{4}\right)} + \underset{④}{\left(\tfrac{3}{4}\right)^2\left(\tfrac{1}{4}\right)}$$

$$\uparrow\ h \quad \uparrow\ b$$

$$= 7/32 = 0.21875$$

Right Endpoints

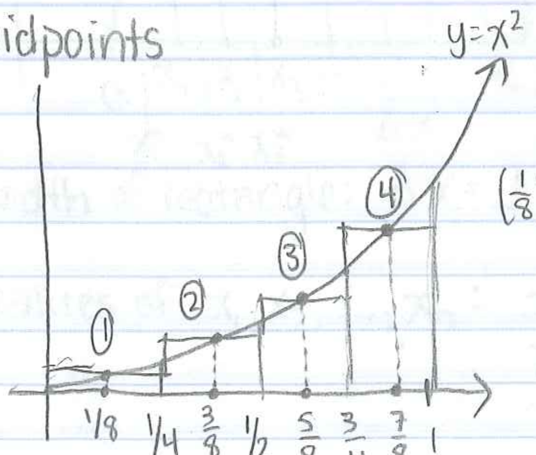


Estimated area:

$$\underset{①}{\left(\tfrac{1}{4}\right)^2\left(\tfrac{1}{4}\right)} + \underset{②}{\left(\tfrac{1}{2}\right)^2\left(\tfrac{1}{4}\right)} + \underset{③}{\left(\tfrac{3}{4}\right)^2\left(\tfrac{1}{4}\right)} + \underset{④}{(1)^2\left(\tfrac{1}{4}\right)}$$

$$= 15/32 = 0.46875$$

Midpoints



Estimated area:

$$\left(\tfrac{1}{8}\right)^2\left(\tfrac{1}{4}\right) + \left(\tfrac{3}{8}\right)^2\left(\tfrac{1}{4}\right) + \left(\tfrac{5}{8}\right)^2\left(\tfrac{1}{4}\right) + \left(\tfrac{7}{8}\right)^2\left(\tfrac{1}{4}\right)$$

$$= 21/64 = .328125$$

How good are our estimates?
- Left is underestimate (in this case)
- Right is overestimate
- midpoint is ambiguous

you must look at the graph to determine over/underestimates

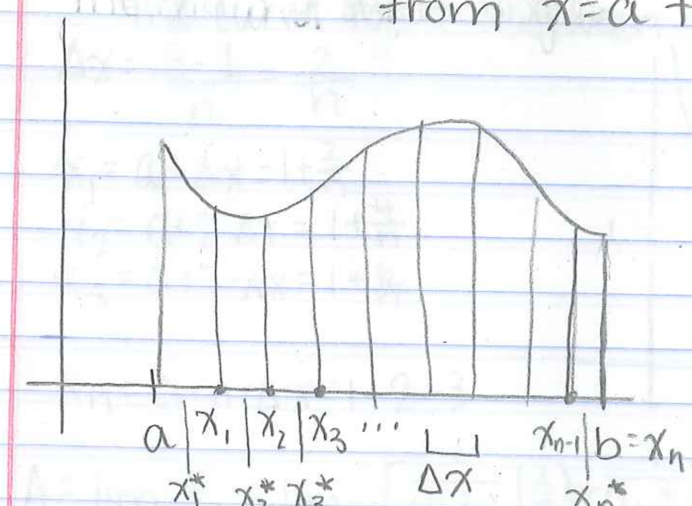
How can we obtain better estimates? $\boxed{\text{Applet}}$
- use more rectangles (INFINITELY MORE)

R_n = approx area using n rectangles and right endpts
L_n = " " left "
M_n = " " midpoints

DEF 1:

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} L_n = \lim_{n\to\infty} M_n$$

Some notation: Calc. area under curve $f(x)$ from $x=a$ to $x=b$ with n rect.



- width of rectangle: $\Delta x = \dfrac{b-a}{n}$
- values of $x_1, x_2, \ldots, x_n$: $x_1 = a + \Delta x$
  $x_2 = a + 2\cdot\Delta x$
  $x_3 = a + 3\cdot\Delta x$
  $\vdots$

DEF 2: The area A of a region that lies under the graph of the continuous function f is the limit of the sum of the areas of approximating rectangles

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty}\left[f(x_1)\Delta x + \underbrace{f(x_2)}_{\text{height}}\underbrace{\Delta x}_{\text{width}} + \dots + f(x_n)\Delta x\right]$$

Sometimes you'll see

$$A = \lim_{n\to\infty}\left[f(x_1^*)\Delta x + f(x_2^*)\Delta x + \dots + f(x_n^*)\Delta x\right]$$

x_i^* are sample points. Just some value between x_{i-1} and x_i. Not nec an endpt.

As long as you use a height the graph takes in a given strip, your rectangles will approx.

Example: Write an expression for the area under the curve $f(x) = \frac{1}{x}$ between $x=1$ and $x=3$

$\Delta x = \frac{3-1}{n} = \frac{2}{n}$

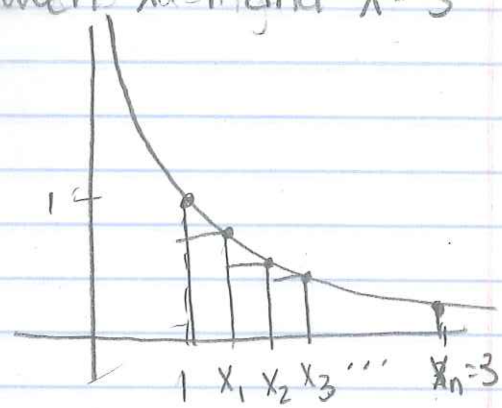
$x_1 = a + \Delta x = 1 + \frac{2}{n}$

$x_2 = a + 2\cdot\Delta x = 1 + \frac{4}{n}$

$x_3 = a + 3\cdot\Delta x = 1 + \frac{6}{n}$

$\vdots$

$x_n = a + n\cdot\Delta x = 1 + 2 = 3$



$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty}\left[\frac{1}{(1+\frac{2}{n})}\left(\frac{2}{n}\right) + \frac{1}{(1+\frac{4}{n})}\left(\frac{2}{n}\right) + \frac{1}{(1+\frac{6}{n})}\left(\frac{2}{n}\right) + \dots + \frac{1}{3}\left(\frac{2}{n}\right)\right]$$

Now, using 6 rectangles, estimate the area under $f(x)=\frac{1}{x}$ from $x=1$ to $x=3$ using:

(a) Right Endpoints

$$\Delta x = \frac{3-1}{6} = \frac{1}{3} \quad R_6 = \left(\frac{1}{1+\frac{1}{3}}\right)\cdot\frac{1}{3} + \left(\frac{1}{1+\frac{2}{3}}\right)\left(\frac{1}{3}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{3}\right) + \left(\frac{1}{1+\frac{4}{3}}\right)\left(\frac{1}{3}\right) + \left(\frac{1}{1+\frac{5}{3}}\right)\left(\frac{1}{3}\right) + \left(\frac{1}{3}\right)\left(\frac{1}{3}\right)$$

$$= .995635$$

(b) Left Endpoints

$$L_6 = 1\cdot\frac{1}{3} + \left(\frac{1}{1+\frac{1}{3}}\right)\cdot\frac{1}{3} + \left(\frac{1}{1+\frac{2}{3}}\right)\left(\frac{1}{3}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{3}\right) + \left(\frac{1}{1+\frac{4}{3}}\right)\left(\frac{1}{3}\right) + \left(\frac{1}{1+\frac{5}{3}}\right)\left(\frac{1}{3}\right)$$

$$= 1.21786$$

The Great Gorilla Jump

A gorilla (wearing a parachute) jumped off of the top of a building. We were able to record the velocity of the gorilla with respect to time once each second. The data is shown below. Note that he touched the ground just after 4 seconds.

Time (in seconds)	Velocity (in feet per second)
0	0
0.5	5
1	7
1.5	8
2	11
2.5	11.5
3	12
3.5	13
4	15.5

1. Approximate how far the gorilla fell during each half second interval and fill in the table below.

	Left	Right	Midpoint
Time interval (in seconds)	Approximate distance traveled		
0 – 0.5	0·(.5)	5·(.5)	(2.5)·(.5)
0.5 – 1.0	5·(.5)	7·(.5)	(6)·(.5)
1.0 – 1.5	7·(.5)	8·(.5)	(7.5)·(.5)
1.5 – 2.0	8·(.5)	11·(.5)	(9.5)·(.5)
2.0 – 2.5	11·(.5)	11.5·(.5)	(11.25)·(.5)
2.5 – 3.0	(11.5)·(.5)	12·(.5)	(11.75)·(.5)
3.0 – 3.5	12·(.5)	13·(.5)	(12.5)·(.5)
3.5 – 4.0	13·(.5)	(15.5)·(.5)	(14.25)·(.5)

2. Approximate the total distance the gorilla fell from the time he jumped off the building until the time he landed on the ground.

L= 33.75 R=41.5 M=37.625

3. Is your approximate an overestimate, and underestimate, or is it the exact value? How can you tell? (you may want to try your hand at graphing…)

L over R under M ambiguous