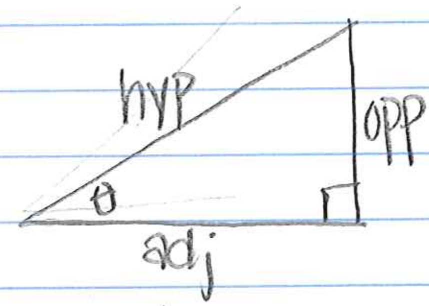
# Trig Review

## The Right Triangle:




The big 3

$\sin\theta = \dfrac{opp}{hyp}$

$\cos\theta = \dfrac{adj}{hyp}$

$\tan\theta = \dfrac{opp}{adj}$

SOH CAH TOA

S - sine, O - opp, H - hyp; C - cos, A - adj, H - hyp; T - tan, O - opp, A - adj

$\sec\theta = \dfrac{hyp}{adj}$

$\csc\theta = \dfrac{hyp}{opp}$

$\cot\theta = \dfrac{adj}{opp}$

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## Everything in terms of sin, cos:

$\sec\theta = \dfrac{1}{\cos\theta}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} \left( = \dfrac{opp/hyp}{adp/hyp} = \dfrac{opp}{adj} \right)$

$\csc\theta = \dfrac{1}{\sin\theta}$

$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$

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## The Biggie: $\sin^2\theta + \cos^2\theta = 1$

Why?

$\sin^2\theta + \cos^2\theta = \left(\dfrac{opp}{hyp}\right)^2 + \left(\dfrac{adj}{hyp}\right)^2 = \dfrac{(opp)^2 + (adj)^2}{(hyp)^2}$

By pythagoras $(opp)^2 + (adj)^2 = (hyp)^2$

so

$\dfrac{(opp)^2 + (adj)^2}{(hyp)^2} = \dfrac{(hyp)^2}{(hyp)^2} = 1 \quad \square$

## Examples

(1) $3\sin^2\theta + 3\cos^2\theta = 3$

(2) $r\sin^2\theta + r\cos^2\theta = r$

(3) $\sin\theta\cos^2\theta + \sin^3\theta = \sin\theta(\sin^2\theta + \cos^2\theta) = \sin\theta$

(4)

## Other Biggies

$\sin^2\theta + \cos^2\theta = 1 \Longrightarrow$

- $1 - \sin^2\theta = \cos^2\theta$
- $1 - \cos^2\theta = \sin^2\theta$
- $1 + \tan^2\theta = \sec^2\theta$

  because:

  $$1 + \tan^2\theta = 1 + \frac{\sin^2\theta}{\cos^2\theta} = \frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta} = \sec^2\theta$$
- $1 + \cot^2\theta = \csc^2\theta$ (same proof)

## Examples:

(1) Simplify the fraction:

$$\frac{1+\tan^2\theta}{1+\cot^2\theta} = \frac{\sec^2\theta}{\csc^2\theta} = \frac{1/\cos^2\theta}{1/\sin^2\theta} = \frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta$$

## Useful-Double Angle Formulas

- $\sin(2\theta) = 2\sin\theta\cos\theta$
- $\cos(2\theta) = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$
- $\tan(2\theta) = \dfrac{2\tan\theta}{1-\tan^2\theta}$

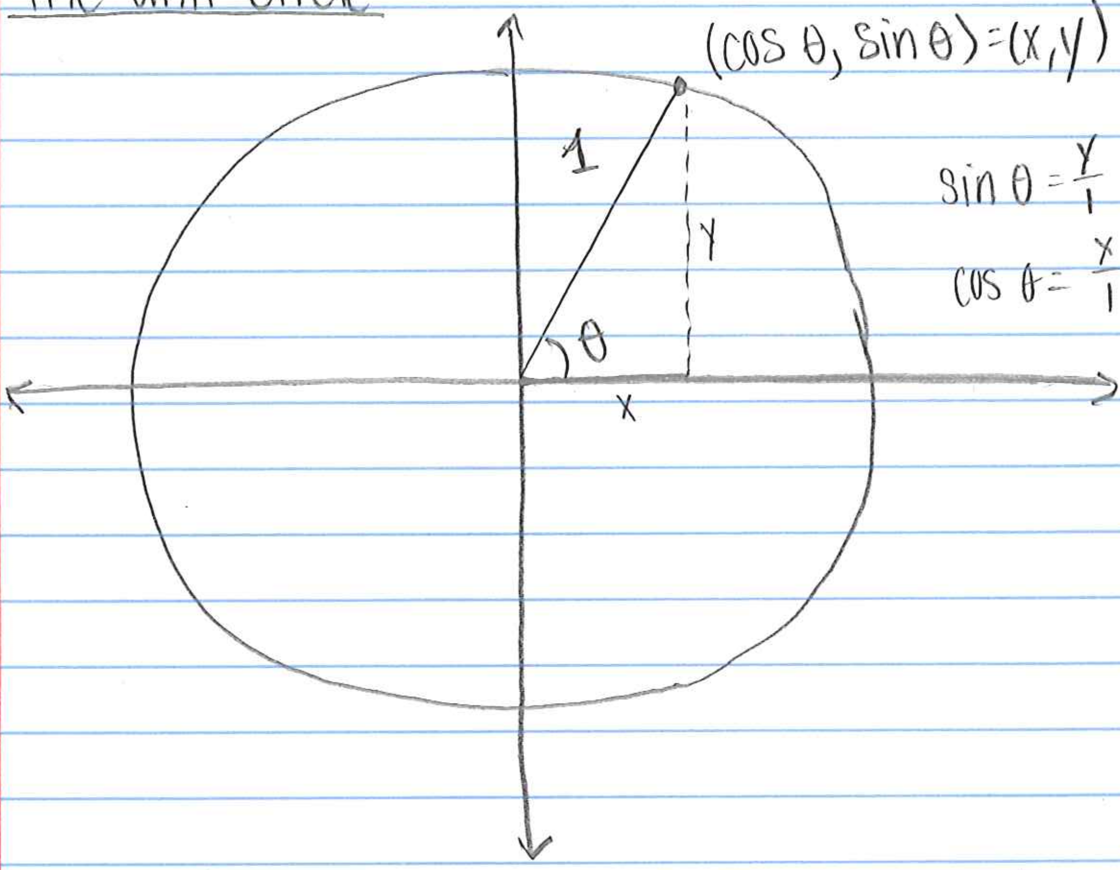
Examples: Simplify the fractions

(1) $$\frac{\sin 2\theta}{1+\cos 2\theta} = \frac{2\sin\theta\cos\theta}{1+(\underbrace{2\cos^2\theta-1}_{\text{get the 1 to cancel out.}})} = \frac{2\sin\theta\cos\theta}{2\cos^2\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

(2) $$\frac{\sin 2\theta}{1-\cos^2\theta} = \frac{2\sin\theta\cos\theta}{\sin^2\theta} = \frac{2\cos\theta}{\sin\theta} = 2\cot\theta$$

$\sin^2\theta + \cos^2\theta = 1$ looks a lot like $y^2 + x^2 = 1$

The Unit Circle

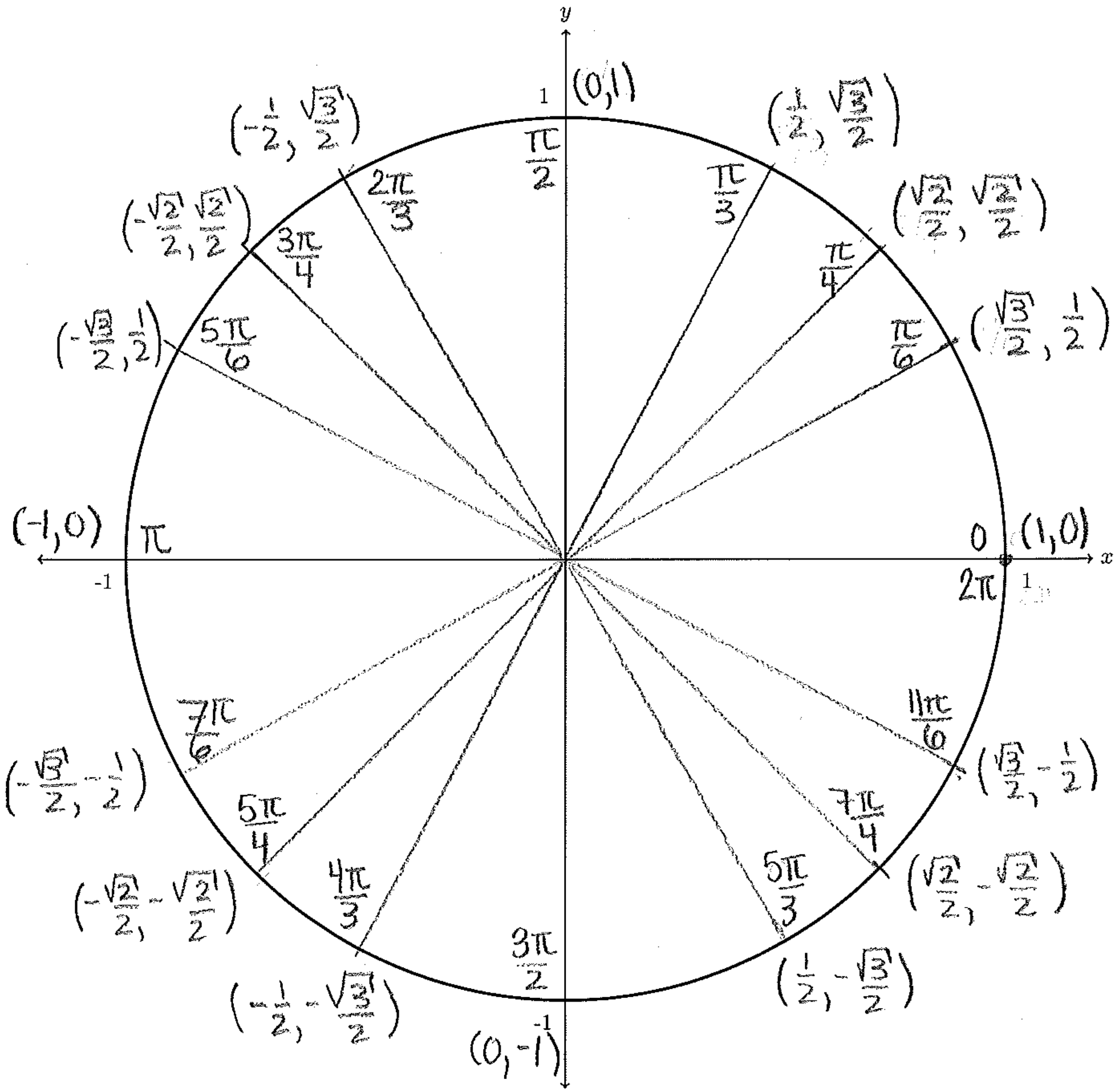



The deal: Look at a point on the unit circle that is $\theta$ radians counterclockwise from pos. x-axis. The coordinates of this point are $(\cos\theta, \sin\theta)$.

* Pass out unit circles, and fill them out

# The Unit Circle

A point on the unit circle that is $\theta$ radian s from pos. $x$-axis is $(\cos\theta, \sin\theta)$.



$\sin^{-1}x, \cos^{-1}x, \tan^{-1}x$ (arcsin x, arccos x, arctan x)

$\sin^{-1}x = y$ means that $\sin y = x$ $\quad -\frac{\pi}{2} \le y \le \frac{\pi}{2}$

$\sin^{-1}x$ only gives values between $-\pi/2$ and $\pi/2$

$\cos^{-1}x = y$ means that $\cos y = x$

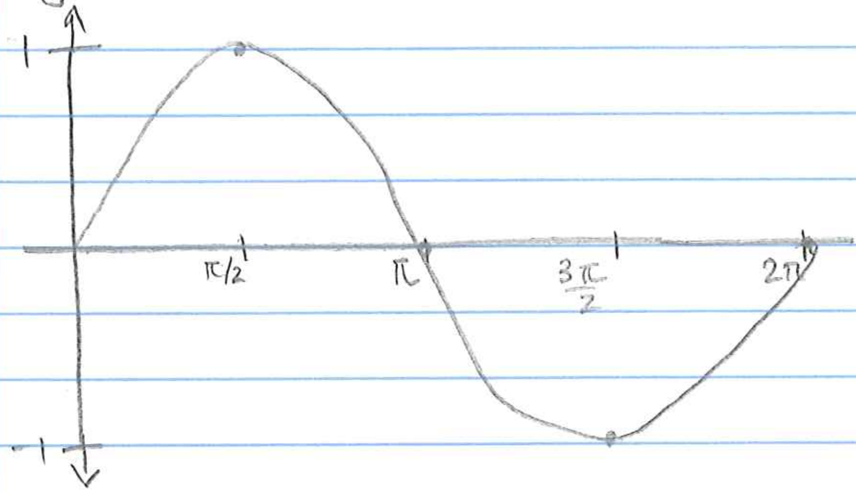
$0 \le y \le \pi$

$\tan^{-1}x = y$ means that $\tan y = x$

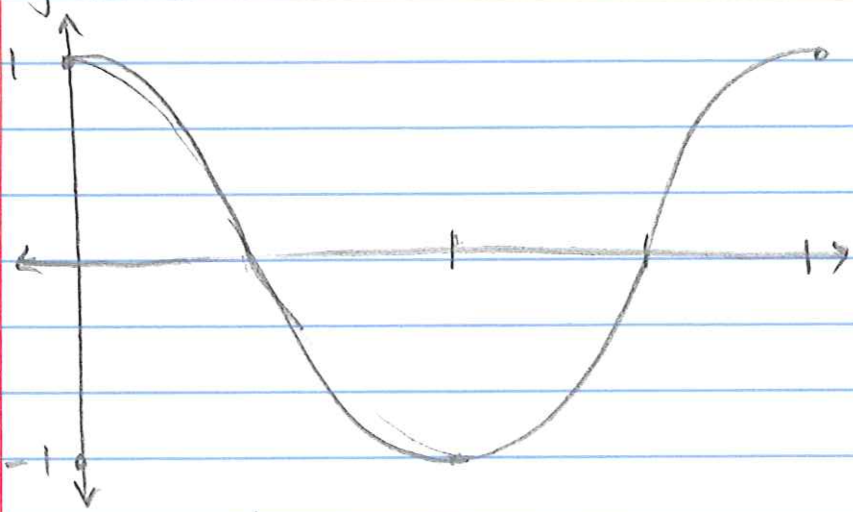
$-\pi/2 < y < \pi/2$

# Graphs

y=sin x




y=cos x




y=tan x