Jan. 30, 2013

Announcements:

- X-hour tomorrow (trig review)
- 6.1 Areas Under Curves due Fri, 2-1
  6.2 Volume is due 2-4 (Mon)
- HW3 due Friday

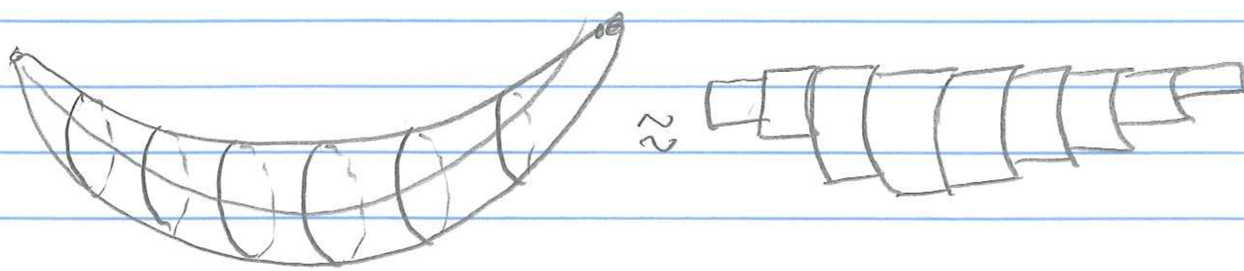
New O.H.'s: (Mostly the same)

M 4-5

W 1-2

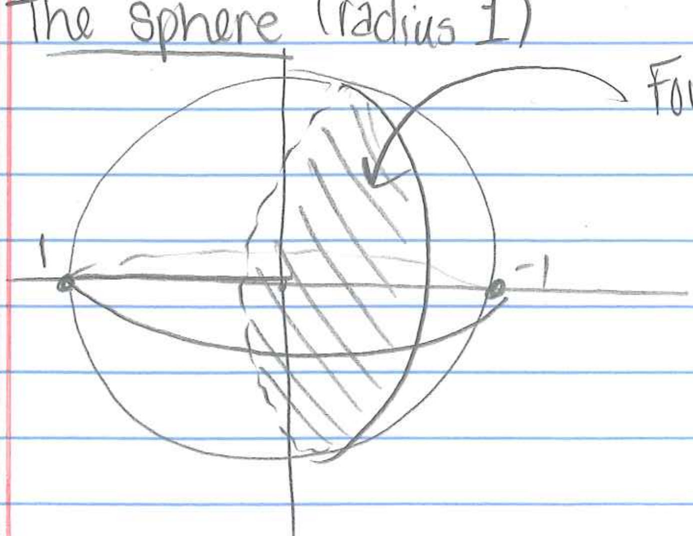
Th 9-11, 2:15-3:15

Last Time: Volume



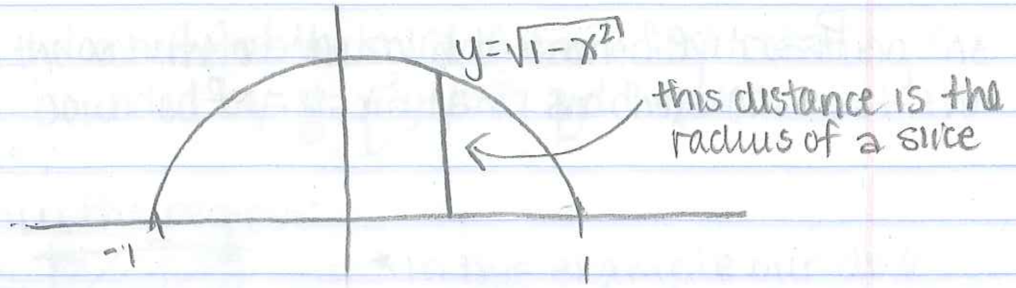
We break a solid up into cylinders,
Consider the volume of # cylinders $\to \infty$

The sphere (radius 1)



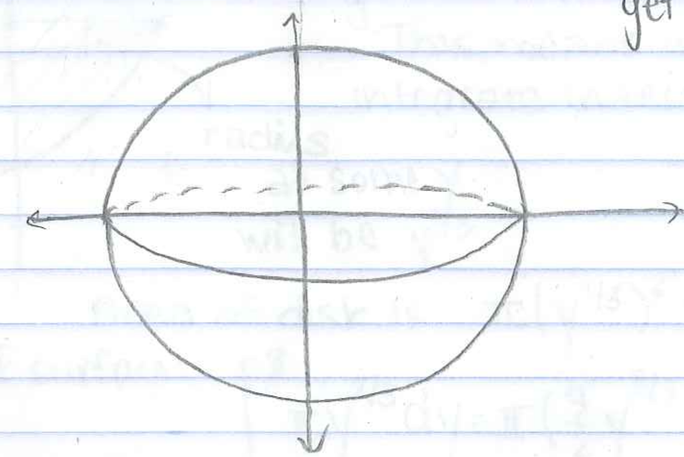
Found area of a slice through x

$$\int_{-1}^{1} A(x)\,dx$$

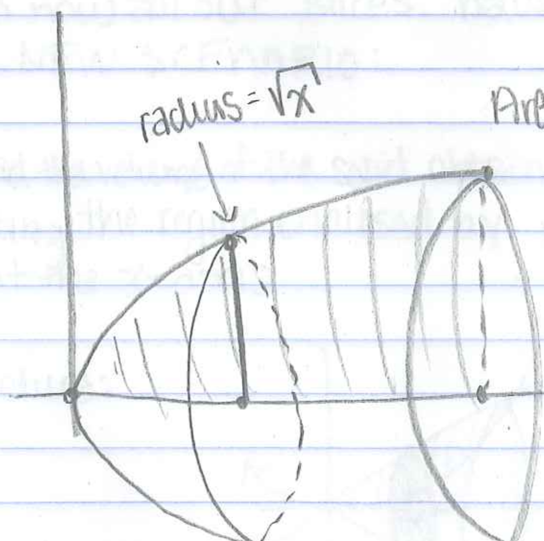


rotate

get a sphere



Example: (2) Find the volume of the solid obtained by rotating about the x-axis the region under the curve $y=\sqrt{x}$ from 0 to 1.



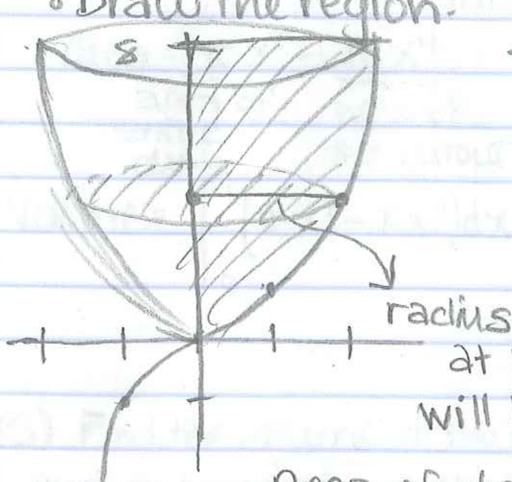
Area of a slice: $\pi(\sqrt{x})^2$
$= \pi x$

Volume $=$

$$\int_0^1 \pi x\,dx = \pi \frac{x^2}{2}\Big|_0^1$$
$$= \pi\frac{(1)^2}{2} - \pi\frac{(0)^2}{2}$$
$$= \boxed{\frac{\pi}{2}}$$

(3) Find the volume of the solid obtained by rotating the region bounded by $y=x^3$, $y=8$, and $x=0$ about the x-axis.

- Draw the region:



* In this example our disks are perpendicular to the y-axis instead of the x-axis. This means we will integrate in terms of y.

Area of disk is $\pi(y^{1/3})^2 = \pi y^{2/3}$

Volume of surface: $\int_0^8 \pi y^{2/3}\,dy = \pi\left(\frac{3}{2}y^{5/3}\right)\Big|_0^8$

$= \pi\left(\frac{3}{5}(8)^{5/3} - \frac{3}{5}(0)^{5/3}\right)$

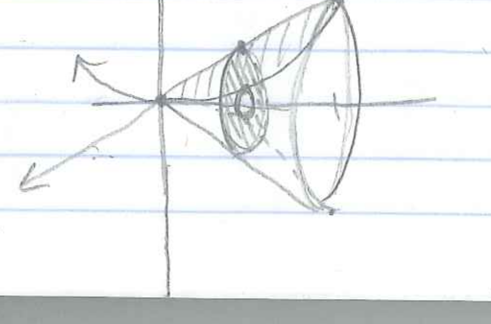
$= \pi \cdot \frac{3}{5} \cdot 32 = \boxed{\frac{96}{5}\pi}$

* Up to now all our "slices" have been disks.

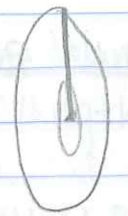
NEW SCENARIO:

(4) Find the volume of the solid obtained by rotating the region enclosed by $y=x$ and $y=x^2$ about the x-axis

The picture:



Before a slice was the shape of a disk, now it has a hole.



The radius of the entire disk is the larger function
radius of the cutout is the smaller function

Radius(larger) $= x$

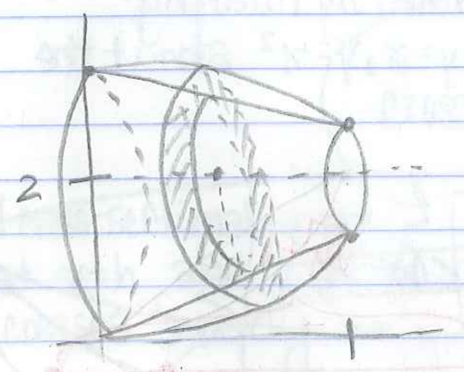
(smaller) $= x^2$

Area $= \underbrace{\pi x^2}_{\text{area of entire disk}} - \underbrace{\pi x^4}_{\text{area of the cutout}}$

$$\text{Volume} = \int_0^1 (\pi x^2 - \pi x^4)\,dx = \pi \frac{x^3}{3} - \pi \frac{x^5}{5} \Big|_0^1$$

$$= \frac{\pi}{3} - \frac{\pi}{5} = \boxed{\frac{2\pi}{15}}$$

(5) Find the volume of the region enclosed by $y = x$, $y = x^2$ and rotated about the line $y = 2$.



Area of washer

larger radius: $2 - x^2$

smaller radius: $2 - x$

Area: $\pi(2-x^2)^2 - \pi(2-x)^2$

$= \pi(4 - 4x^2 + x^4 - 4 + 4x - x^2)$

$= \pi(x^4 - 5x^2 + 4x)$

$$\text{Volume: } \int_0^1 \pi(x^4 - 5x^2 + 4x)\,dx = \pi\left(\frac{x^5}{5} - \frac{5}{3}x^3 + \frac{4x^2}{2}\right)\Big|_0^1$$

$$= \pi\left(\frac{1}{5} - \frac{5}{3} + \frac{1}{2}\right) = \frac{8\pi}{15}$$