Jan. 28, 2013

Announcements:

- Midterm tomorrow, 7-9 PM (Wilder 111)
  (No calc, Notes, Textbook); ch. 5
- O.H. today 4-5, Tues 7:45-12
- 6.1 Areas Under Curves is open, due 2-1
  6.2 Volume is due 2-4
- HW3 posted, due Friday

Last Time: Areas Between Curves

Example: (1) Area enclosed by $y = (x^2 - 2x)$

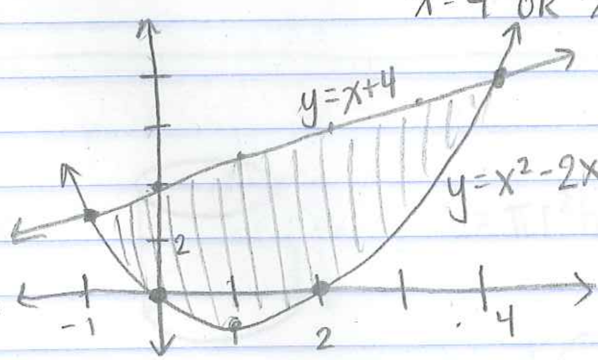
$y = x + 4$

Things to do: Draw a picture (test some pts)
Find Intersection points
Setup integral.

Intersection pts: $x^2 - 2x = x + 4$

$x^2 - 3x - 4 = 0 \Rightarrow (x-4)(x+1)$

$x = 4$ OR $x = -1$



$$\int_{-1}^{4} (x+4) - (x^2 - 2x)\, dx = \int_{-1}^{4} 3x - x^2 + 4\, dx = \frac{3}{2}x^2 - \frac{x^3}{3} + 4x \Big|_{-1}^{4}$$

$$= \frac{3}{2}(4)^2 - \frac{(4)^3}{3} + 4 \cdot 4$$

$$- \frac{3}{2}(-1)^2 - \frac{(-1)^3}{3} + 4(-1)$$

$$= 24 - \frac{64}{3} + 16 + \frac{3}{2} + \frac{1}{3} - 4$$

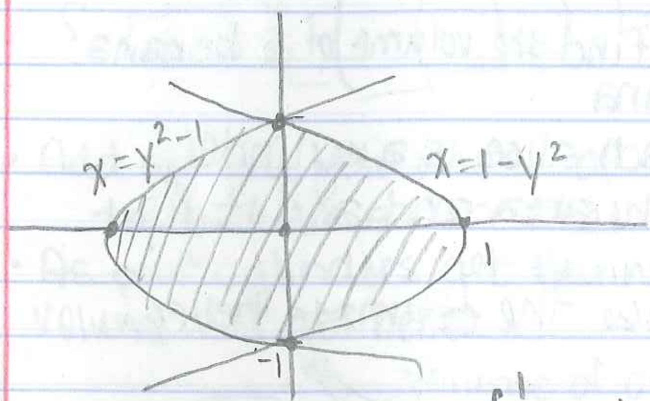
(2) $x=1-y^2$, $x=y^2-1$

Intersection points: $1-y^2=y^2-1$

$2y^2-2=0$

$2(y^2-1)=0 \Leftrightarrow 2(y+1)(y-1)=0$

$y=-1$ OR $y=1$



Picture: know these are both sideways parabolas. Find pts w/ $y=1,0,-1$

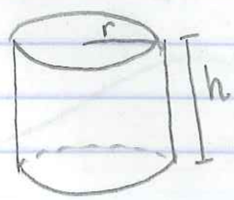
$$\int_{-1}^{1}(1-y^2)-(y^2-1)\,dy=\int_{-1}^{1}2-2y^2\,dy$$

$$=2y-\frac{2}{3}y^3\Big|_{-1}^{1}=2-\frac{2}{3}-2(-1)+\frac{2}{3}(-1)^3$$

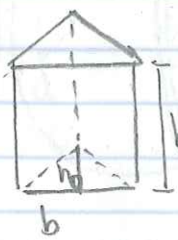
$$=2-\frac{2}{3}+2-\frac{2}{3}=4-\frac{4}{3}=\frac{8}{3}$$

<u>Volume</u>

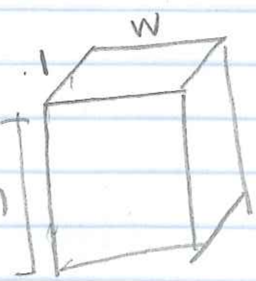
The cylinder:



$V=\pi r^2 h$

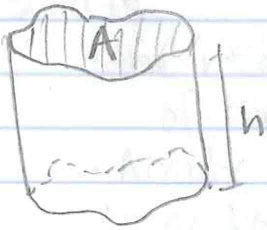


$V=\frac{1}{2}bh_0\cdot h$



$V=l\cdot w\cdot h$

In general:



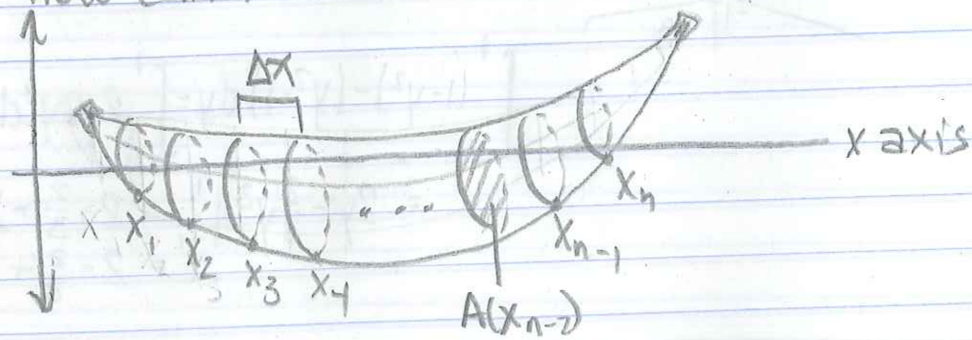
$V=Ah$

The Volume of a Banana

How would I find the volume of a banana?

- Sliced banana
- pretend each slice is a cylinder, calculate the area and add it up.

• How can I make the estimate better?

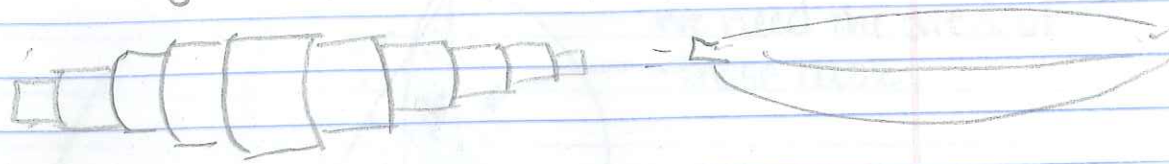


Volume:

$$\lim_{n \to \infty} \left[A(x_1)\Delta x + A(x_2)\Delta x + A(x_3)\Delta x + \dots + A(x_n)\Delta x \right]$$

Banana: How could we find the volume
- thin slices are almost cylinders.

Pretend the banana is made up of cylinders.



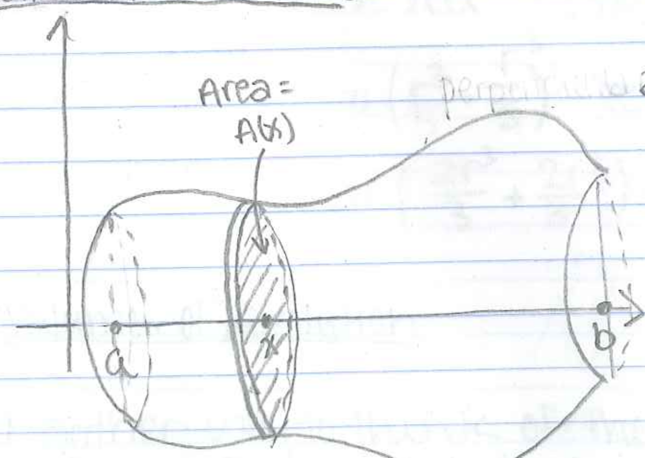
- Add up the volumes of all these cylinders to get an estimate.
- As our cylinders get thinner, our estimation of the volume will get better.

Volume of one of our cylinders

$\underbrace{A}_{\text{area of the face.}} \underbrace{\Delta x}_{\text{"height"}}$

We want to consider this sum as $n \to \infty$

<u>Definition of Volume:</u>



$A(x)$ gives the area of a a disk perpendicular to the x axis, that goes through x.

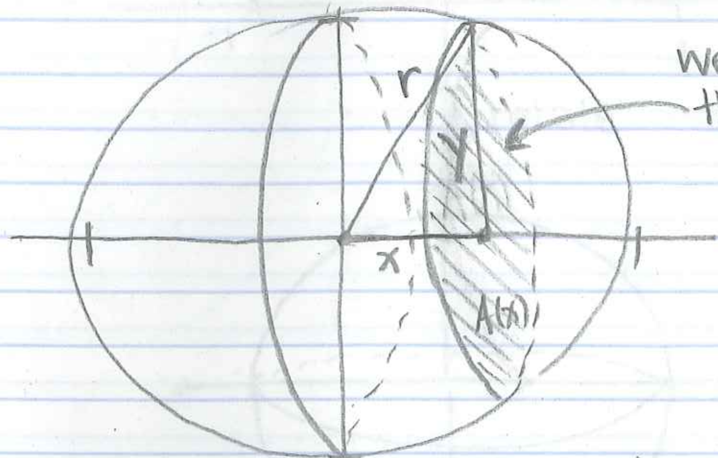
The volume of this solid is

$$\int_a^b A(x)\,dx$$

- the integral adds up the "volumes" of the disks
- $A(x)dx$ is "volume" of each disk $\approx A\Delta x$

Examples: (1) Show that the volume of a sphere of radius r is $V=\frac{4}{3}\pi r^3$.

- Put a sphere on the coordinate axes: (centered about the origin)



we need the area of these disks

Need y in terms of x (and r)

$x^2+y^2=r^2$ so $\sqrt{r^2-x^2}=y$ (this works since y is a distance in this situation)

So $A(x)=\pi\left(\sqrt{r^2-x^2}\right)^2$
$=\pi(r^2-x^2)$

Now $V=\int_{-r}^{r}\pi(r^2-x^2)\,dx=\pi\left(r^2x-\frac{x^3}{3}\right)\Big|_{-1}^{1}$

$=\pi\left(r^3-\frac{r^3}{3}\right)-\pi\left(-r^3-\frac{(-r)^3}{3}\right)$

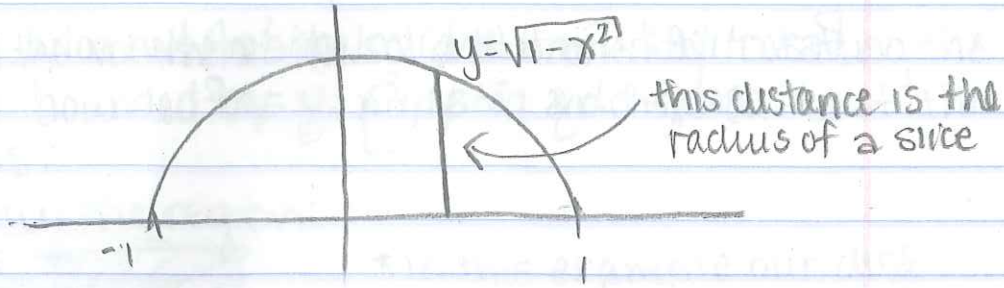
$=\pi\left(\frac{2r^3}{3}+\frac{2r^3}{3}\right)=\frac{4}{3}\pi r^3$

Volumes of Revolution

Another way to think of the sphere:

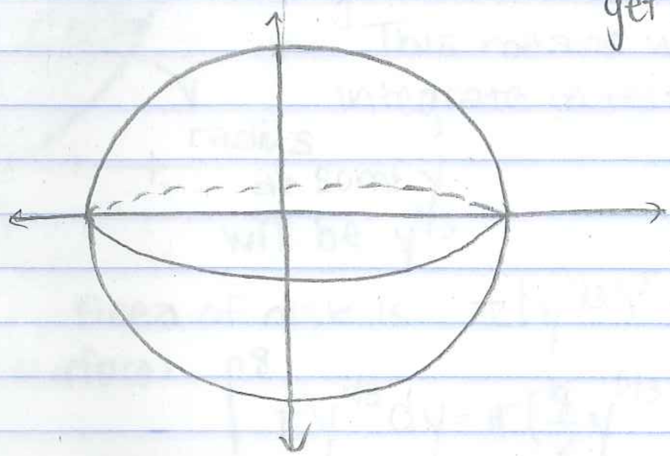
Sphere of radius 1 ($r=1$):

Find the volume of the solid obtained by rotating $y=\sqrt{1-x^2}$ around the x-axis.

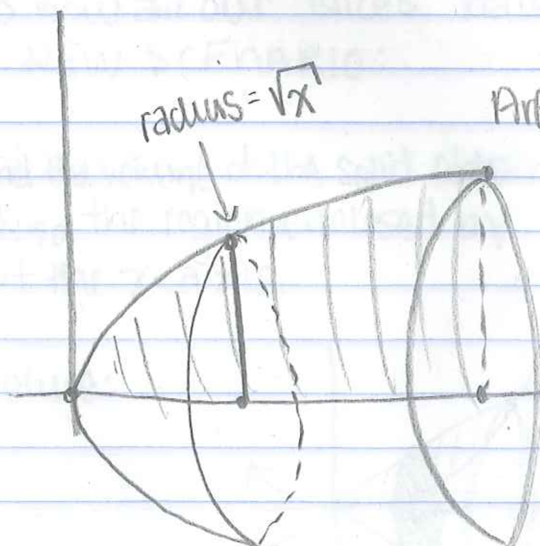


rotate

get a sphere



<u>Example</u>: (2) Find the volume of the solid obtained by rotating about the x-axis the region under the curve $y=\sqrt{x}$ from 0 to 1.



Area of a slice: $\pi(\sqrt{x})^2$
$= \pi x$

Volume =

$$\int_0^1 \pi x \, dx = \pi \frac{x^2}{2} \Big|_0^1$$

$$= \pi \frac{(1)^2}{2} - \pi \frac{(0)^2}{2}$$

$$= \boxed{\frac{\pi}{2}}$$