Jan. 23, 2013

Announcements:

- X-hour tomorrow
- HW2 Due Friday
- Midterm 1 on Tues, Jan 29 in Wilder 111
  Covers ALL of chapter 5
- 5.5 Substitution Rule is due Monday (1/28)

We left off at u-substitution:

Example

(1) $\int e^{5x}\,dx = ?$

It looks like there is a function inside, so we may want to try substitution.

$u = 5x$

$du = 5dx$

$$\Rightarrow \int e^{5x} \underbrace{dx}_{\frac{du}{5}} = \int e^{u} \frac{du}{5} = \frac{1}{5}\int e^{u}du$$

$$= \frac{1}{5}e^{u} + C$$

$$= \boxed{\frac{1}{5}e^{5x} + C}$$

(2) $\int x^3 e^{x^4}\,dx$ $\quad u = x^4$

$du = 4x^3 dx$

$=$

$$\int \frac{1}{4} e^{u} du = \frac{1}{4}e^{u} = \boxed{\frac{1}{4}e^{x^4}}$$

What???

This process is the opposite of the chain rule

Let's check it: outside

$$\frac{d}{dx}\left(\frac{2}{3}\underbrace{(1+x^2)}_{\text{inside}}^{3/2}+C\right)=\left(\frac{2}{3}\right)\left(\frac{3}{2}\right)(1+x^2)^{1/2}(2x)$$

$$=2x\sqrt{1+x^2} \checkmark$$

Examples: (1) $\int x^3\cos(x^4+2)\,dx$

↳ bleh, make it go away

$u=x^4+2$

$\frac{du}{dx}=4x^3 \Rightarrow du=4x^3\,dx$

$$\int \cos(\underbrace{x^4+2}_{u})\cdot \underbrace{x^3\,dx}_{\frac{du}{4}}=\int \cos u\cdot\frac{du}{4}=\frac{1}{4}\int \cos u\,du$$

$$=\frac{1}{4}\sin u+C$$

$$=\frac{1}{4}\sin(x^4+2)+c$$

The process for u-substitution (Indefinite integrals)

(1) decide possibilities for what u equals, choose one

(2) calculate du

(3) substitute u & du into your integral, result should be an integral completely in terms of u (no x's)

(4) take resulting integral

(5) unsubstitute (put the x's back in)

Try it out: $\int\sqrt{2x+1}\,dx$ let $u=2x+1$

$du=2\,dx$

$$=\int\sqrt{u}\,\frac{du}{2}=\frac{u^{3/2}}{3/2}\cdot\frac{1}{2}=\frac{u^{3/2}}{3}$$

Substitution and Definite Integrals

$$\int_0^4 \sqrt{2x+1}\, dx \qquad u = 2x+1$$
$$du = 2dx$$

Method 1

$$\int_{x=0}^{x=4} \sqrt{u}\, \frac{du}{2}$$

$$= \frac{2u^{3/2}}{2\cdot 3}\Bigg|_{x=0}^{x=4}$$

* * * *
Must put x back in before evaluating at the bounds
* * * *

$$= \frac{(2x+1)^{3/2}}{3}\Bigg|_0^4$$

$$= \frac{9^{3/2}}{3} - \frac{1^{3/2}}{3} = \boxed{\frac{26}{3}}$$

Method 2:

$$\int_{u(0)}^{u(4)} \sqrt{u}\, \frac{du}{2}$$

$$= \int_1^9 \frac{1}{2}\sqrt{u}\, du$$

$$= \frac{2\cdot u^{3/2}}{2\cdot 3}\Bigg|_1^9 = \frac{9^{3/2}}{3} - \frac{1^{3/2}}{3}$$

* In this method you don't substitute the x back in.

Example: (1) $\int_1^e \frac{\ln x}{x}\, dx \qquad u = \ln x$, $du = \frac{dx}{x}$

$$= \int_{x=1}^{x=e} u\, du$$

$$= \frac{u^2}{2}\Bigg|_{x=1}^{x=e} = \frac{(\ln x)^2}{2}\Bigg|_1^e = \frac{(\ln e)^2}{2} - \frac{(\ln 1)^2}{2}$$

$$= \boxed{\frac{1}{2}}$$

(2) $$\int_1^2 \frac{dx}{(3-5x)^2} \qquad u=3-5x$$
$$du=-5\,dx \Longleftrightarrow \frac{du}{-5}=dx$$

$$=\int_{x=1}^{x=2} \frac{1}{u^2}\cdot\frac{du}{-5} = \int_{x=1}^{x=2} \left(-\frac{1}{5}\right)(u^{-2})\,du$$

$$= \left(-\frac{1}{5}\right)\frac{u^{-1}}{-1}\Bigg|_{x=1}^{x=2}$$

$$= \frac{1}{5u}\Bigg|_{x=1}^{x=2} = \frac{1}{5(3-5x)}\Bigg|_1^2$$

$$= \frac{1}{5(3-5\cdot 2)} - \frac{1}{5(3-5\cdot 1)}$$

$$= \frac{1}{-35} + \frac{1}{10} = \frac{1}{14}$$

(3) (Trickier)

$$\int \sqrt{1+x^2}\,\underbrace{x^5\,dx}_{\text{what do we do about this?}} \qquad u=1+x^2, \quad du=2x\,dx$$

$$=\int \underbrace{\sqrt{1+x^2}}_{\sqrt{u}}\cdot\underbrace{x^4}_{?}\cdot\underbrace{x\cdot dx}_{\frac{du}{2}}$$

Write x^4 in terms of u

$u-1=x^2$

$(u-1)^2=x^4$

Expand

$$=\int \sqrt{u}\cdot(u-1)^2\frac{du}{2} = \frac{1}{2}\int\sqrt{u}(u^2-2u+1)\,du = \frac{1}{2}\int(u^{5/2}-2u^{3/2}+u^{1/2})\,du$$

$$= \frac{1}{2}\left(\frac{2u^{7/2}}{7} - \frac{2\cdot 2u^{5/2}}{5} + \frac{2u^{3/2}}{3}\right) + C = \frac{(1+x^2)^{7/2}}{7} - \frac{2(1+x^2)^{5/2}}{5} + \frac{(1+x^2)^{3/2}}{3} + C$$

Substitution Practice

1. $\int_0^\pi \sin(x/3)\, dx$

$u = x/3$

$du = \frac{1}{3}dx \iff 3du = dx$

$$\int_0^\pi \sin\frac{x}{3}\, dx = \int_{x=0}^{x=\pi} \sin u \cdot 3du = 3(-\cos u)\Big|_{x=0}^{x=\pi} = -3\cos\frac{x}{3}\Big|_0^\pi$$

$$= -3\cos\frac{\pi}{3} + 3\cos(0)$$

$$= -\frac{3}{2} + 3 = \boxed{\frac{3}{2}}$$

2. $\int_{-1}^0 x(x^2+2)^2\, dx$ $\quad u = x^2+2 \quad du = 2x\,dx$

$$= \int_{x=-1}^{x=0} u^2 \frac{du}{2} = \frac{u^3}{2\cdot 3}\Big|_{x=-1}^{x=0}$$

$$= \frac{(x^2+2)^3}{6}\Big|_{-1}^0 = \frac{(0+2)^3}{6} - \frac{((-1)^2+2)^3}{6} = \frac{8}{6} - \frac{27}{6} = \boxed{-\frac{19}{6}}$$

3. $\int_1^2 \frac{z}{3+z^2}\, dz$

$u = 3+z^2$

$du = 2z\,dz$

$$\int_1^2 \frac{z}{3+z^2}\, dz = \int_{z=1}^{z=2} \frac{1}{u}\frac{du}{2} = \frac{1}{2}\ln u\Big|_{z=1}^{z=2} = \frac{1}{2}\ln(3+z^2)\Big|_1^2$$

$$= \boxed{\frac{1}{2}(\ln 7 - \ln 4)}$$

4. $\int_0^1 \frac{e^x}{e^x+4}\, dx$

$u = e^x+4$

$du = e^x dx$

$$\int_0^1 \frac{e^x}{e^x+4}\, dx = \int_{x=0}^{x=1} \frac{1}{u}\, du = \ln u\Big|_{x=0}^{x=1} = \ln(e^x+4)\Big|_0^1$$

$$= \boxed{\ln(e+4) - \ln(5)}$$

5. $\int_{-5}^0 (2x+5)(x^2+5x)^7\, dx$

$u = x^2+5x$

$du = (2x+5)dx$

$$\int_{-5}^0 (2x+5)(x^2+5x)^7\, dx = \int_{x=-5}^{x=0} u^7\, du = \frac{u^8}{8}\Big|_{x=-5}^{x=0}$$

$$= \frac{(x^2+5x)^8}{8}\Big|_{-5}^0$$

$$= 0 - \frac{(25-25)^8}{8} = \boxed{0}$$