Announcements:
- X-hour next week
- Monday class, office hour canceled
- HW1 Due today, HW2 posted
- WeBWork: 5.3 Fundamental Theorem Due Monday (1/21)
  5.4 Indefinite Integral Due Wednesday (1/23)
- Quiz 2 on Wednesday (1/23)
  - covers 5.3, 5.4 ; 15 minutes (8:45 - 9:00)
- Quiz solutions posted on exam page

Last time: FTC and Indefinite integrals

Examples: (1) $\int_1^3 (\ln x + 2x^2)\,dx$

$$= \frac{1}{x} + \frac{2x^3}{3}\Big|_1^3$$

$$= \left(\frac{1}{3} + \frac{2(3)^3}{3}\right) - \left(\frac{1}{1} + \frac{2\cdot 1^3}{3}\right)$$

$$= \frac{1}{3} + 18 - 1 - \frac{2}{3} = 50/3$$

(2) $\int_1^4 \frac{t^2\sqrt{t}-1}{t^2}\,dt = \int_1^4 \left(\sqrt{t} - \frac{1}{t^2}\right) dt$

$$= \int_1^4 (t^{1/2} - t^{-2})\,dt$$

$$= \frac{t^{3/2}}{3/2} - \frac{t^{-1}}{-1}\Big|_1^4$$

$$= \left(\frac{2}{3}(4)^{3/2} + \frac{1}{4}\right) - \left(\frac{2}{3}(1^{3/2}) + 1\right)$$

$$= \frac{16}{3} + \frac{1}{4} - \frac{2}{3} - 1 = 47/12$$

Jan. 18, 2013

(6) $\int_1^2 \left(2x^3 - \frac{5}{x}\right)dx = \left(\frac{2x^4}{4} - 5\cdot\ln x\right)\Big|_1^2$

$= \frac{2\cdot 2^4}{4} - 5\cdot\ln(2) - \left(\frac{2\cdot 1^4}{4} - 5\cdot\ln(1)\right)$

$= 8 - 5\cdot\ln(2) - \frac{1}{2} + 0$

<u>Particles, Velocity, Acceleration</u>

We have a velocity function, $v(t)$ for $a \le t \le b$

We know that $v(t) = s'(t)$ ($s(t)$ is position)

<u>Working Example:</u>

A particle is moving along a straight line. The following is the velocity function for <u>$1 \le t \le 4$</u>

$v(t) = t^2 - t - 6$

What is the displacement?

$s(4) - s(1)$

$= \int_1^4 v(t)dt = \int_1^4 (t^2 - t - 6)dt$

$= \frac{t^3}{3} - \frac{t^2}{2} - 6t \Big|_1^4$

$= \left(\frac{4^3}{3} - \frac{4^2}{2} - 6\cdot 4\right) - \left(\frac{1^3}{3} - \frac{1^2}{2} - 6\cdot 1\right)$

$= -9/2$

Displacement = Net change in Position.

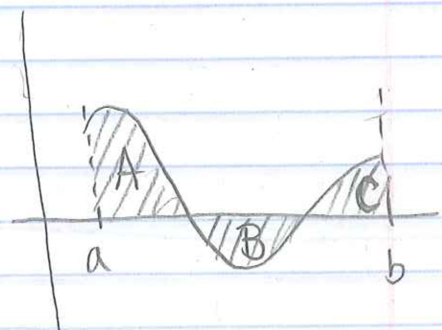
$\int_a^b v(t)dt = s(b) - s(a)$ is Displacement

Total Distance Traveled

*you have to count when backtracking

$$\int_1^4 |t^2 - t - 6|\,dt$$

$$\int_a^b |v(t)|\,dt = \text{Distance Traveled}$$



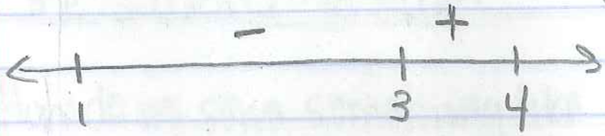
Distance traveled = A+B+C

Displacement = A+C−B

*First calculate where v(t) is negative:

$t^2 - t - 6 = (t-3)(t+2)$

hits x axis when $t=3$, $t=-2$



So,

$$\int_1^4 |t^2-t-6|\,dt = \int_1^3 -(t^2-t-6)\,dt + \int_3^4 (t^2-t-6)\,dt$$

$$= -\left(\frac{t^3}{3} - \frac{t^2}{2} - 6t\right)\Big|_1^3 + \left(\frac{t^3}{3} - \frac{t^2}{2} - 6t\right)\Big|_3^4$$

$$= \frac{61}{6} \approx 10.17 \text{ m}$$

Acceleration:

Acceleration of a particle is $a(t) = t+4$ for $0 \le t \le 10$

We also know that $v(0) = 5$

What is the velocity at time t?

$$\int a(t)\,dt = \int (t+4)\,dt = \frac{t^2}{2} + 4t + C$$

$v'(t) = a(t)$

so

$\int a(t)\,dt$ can help us go backwards *what is C?

Know $v(0)=5$

$v(t)=\frac{t^2}{2}+4t+C$

$v(0)=\frac{0^2}{2}+4\cdot 0+C=5$

$C=5$

$v(t)=\frac{t^2}{2}+4t+5$

The Substitution Rule:

How do we solve something like

$\int 2x\sqrt{1+x^2}\,dx$?

We should introduce a new variable, to simplify the expression.

Let $u=1+x^2$

Notice: $\frac{du}{dx}=2x$

let's think of this as du divided by dx

(even though that's not technically correct)

so,

$dx\cdot\frac{du}{dx}=2x\cdot dx$

$du=2x\cdot dx$

observe, $\int 2x\sqrt{1+x^2}\,dx=\int\sqrt{\underset{u}{1+x^2}}\,\underbrace{2x\cdot dx}_{du}=\int\sqrt{u}\,du$

$=\frac{2}{3}u^{3/2}+C=\frac{2}{3}(1+x^2)^{3/2}+C$

What???

This process is the opposite of the chain rule

Let's check it: outside

$$\frac{d}{dx}\left(\frac{2}{3}(\underbrace{1+x^2}_{\text{inside}})^{3/2}+C\right) = \left(\frac{2}{3}\right)\left(\frac{3}{2}\right)(1+x^2)^{1/2}(2x)$$

$$= 2x\sqrt{1+x^2} \checkmark$$

Examples: (1) $\int x^3 \cos(x^4+2)\,dx$

bleh, make it go away

$u = x^4+2$

$\frac{du}{dx} = 4x^3 \Rightarrow du = 4x^3\,dx$

$$\int \cos(\underbrace{x^4+2}_{u}) \cdot \underbrace{x^3\,dx}_{\frac{du}{4}} = \int \cos u \cdot \frac{du}{4} = \frac{1}{4}\int \cos u\,du$$

$$= \frac{1}{4}\sin u + C$$

$$= \frac{1}{4}\sin(x^4+2) + C$$

The process for u-substitution (Indefinite integrals)

(1) decide possibilities for what u equals, choose one
(2) calculate du
(3) substitute u & du into your integral, result should be an integral completely in terms of u (no x's)
(4) take resulting integral
(5) unsubstitute (put the x's back in)

Try it out: $\int \sqrt{2x+1}\,dx$ let $u = 2x+1$

$du = 2\,dx$

$$= \int \sqrt{u}\,\frac{du}{2} = \frac{u^{3/2}}{3/2}\cdot\frac{1}{2} = \frac{u^{3/2}}{3}$$