Jan 17, 2013

Announcements:

- HW1 is ~~Due~~ tomorrow (1/18)
- WeBWork: 5.2 The Definite Integral is due tomorrow (1/18)
  5.3 Fundamental Theorem is due monday (1/21)

The Fundamental Thm of Calculus

Suppose f is continuous on $[a,b]$

Part 1: If $g(x) = \int_a^x f(t)\,dt$, then $g'(x) = f(x)$

Part 2: $\int_a^b f(x)\,dx = F(b) - F(a)$ where F is an antiderivative of f (so $F' = f$)

1/17/18

Part 2 of the thm follows from part 1, and tells us how to eval. def. integrals:

FTC

THM (Part II): If f is continuous on $[a,b]$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

where F is any antiderivative of f, that is, a function s.t. $F' = f$.

Does that really work?

We know $g(x) = \int_a^x f(t)dt$ is an antiderivative of $f(x)$

$$g(b) - g(a) = \int_a^b f(t)dt - \int_a^a f(t)dt = \int_a^b f(t)dt$$

this is the same as $\int_a^b f(x)dx$.

Examples: (of using FTC part II)

(1) $\int_1^3 e^x dx$ $\quad f(x)=e^x$, $F(x)=e^x$
since $F'(x)=e^x=f(x)$

$= F(3)-F(1)= e^3-e^1$ (notation $F(3)-F(1)=F(x)\Big]_1^3$)

(2) Area under $y=x^2$ from 0 to 1

$f(x)=x^2$
$F(x)=\frac{x^3}{3}$

$A=\int_0^1 x^2 dx$

$= \frac{x^3}{3}\Big]_0^1 = \frac{(1)^3}{3}-\frac{(0)^3}{3}=\frac{1}{3}$ $\quad$ since $F'(x)=\frac{3x^2}{3}=x^2=f(x)$

(3) (practice)
Evaluate $\int_1^3 (x^2-2)\,dx$ using FTC II

$= \int_1^3 x^2 dx - \int_1^3 2\,dx = \frac{x^3}{3}\Big]_1^3 - 2x\Big]_1^3 =$

$\frac{(3)^3}{3}-\frac{(1)^3}{3}-2(3)-(-2\cdot(1))$

$= 9-1/3-6+2=\boxed{\frac{14}{3}}$

The WHOLE Shebang

The Fundamental Thm of Calculus:
Suppose f is continuous on $[a,b]$
1. If $g(x)=\int_a^x f(t)\,dt$, then $g'(x)=f(x)$
2. $\int_a^b f(x)\,dx = F(b)-F(a)$, where $F'=f$

Indefinite Integrals

This is our new name for antiderivatives

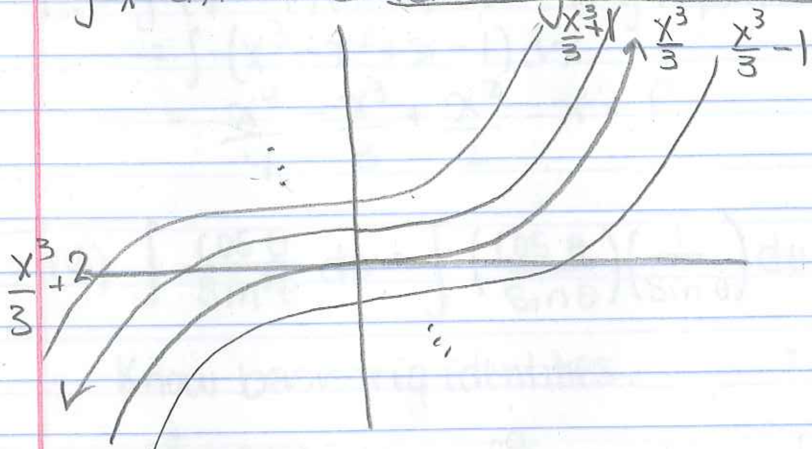
So: $\int f(x)\,dx = F(x)$ means $F'(x) = f(x)$

$\int f(x)\,dx$ is the <u>most general antiderivative</u> (this is where $+C$ comes in)

For example:

$$\int x^2\,dx = \frac{x^3}{3} + C \quad \text{b/c} \quad \frac{d}{dx}\left(\frac{x^3}{3} + C\right) = x^2$$

$\int x^2\,dx$ is a <u>family of functions</u> displace by adding a constant



Moral: When taking indefinite integrals, you must have $+C$ in your answer.

<u>But</u>, when taking definite integrals, $+C$ will <u>not</u> be in your answer (it cancels out)

<u>Indefinite:</u>

$$F(x) = \int x^2\,dx = \frac{x^3}{3} + C$$

<u>Definite:</u>

$$\int_0^1 x^2\,dx = \left(F(x)\right]_0^1 = \frac{x^3}{3} + C\Big]_0^1$$

$$= \left(\frac{1^3}{3} + C\right) - \left(\frac{0^3}{3} + C\right) = \frac{1}{3}$$

We need to know some antiderivatives to compute indefinite integrals:

table on pg. 398

Properties of def Integrals apply [(1)-(4)]

Examples: (1) $\int (3x+\sin x)\,dx = \dfrac{3x^2}{2} + (-\cos x) + C$

CHECK: $\dfrac{d}{dx}\left(\dfrac{3x^2}{2} - \cos x + C\right) = 3x + \sin x$

(2) $\int (x^3 - 4x)\,dx = \dfrac{x^4}{4} - \dfrac{4x^2}{2} + C = x^4/4 - 2x^2 + C$

(3) $\int (x^2+1)(x-1)\,dx$ (Try expanding first)

$= \int (x^3 - x^2 + x - 1)\,dx$

$= \dfrac{x^4}{4} - \dfrac{x^3}{3} + \dfrac{x^2}{2} - x + C$

(4) $\int \dfrac{\cos\theta}{\sin^2\theta}\,d\theta = \int \left(\dfrac{\cos\theta}{\sin\theta}\right)\left(\dfrac{1}{\sin\theta}\right)d\theta = \int \cot\theta \csc\theta \,d\theta$

Know basic trig identities. $= -\csc\theta + C$

(5) $\int_1^9 \dfrac{t^2\sqrt{t} - 1}{t^2}\,dt = \int_1^9 \left(\sqrt{t} - \dfrac{1}{t^2}\right)dt = \int_1^9 \left(t^{1/2} - t^{-2}\right)dt$

$= \dfrac{t^{3/2}}{3/2} - \dfrac{t^{-1}}{-1}\Big|_1^9 = \left(\dfrac{9^{3/2}}{3/2} + 9^{-1}\right) - \left(\dfrac{1^{3/2}}{3/2} + 1^{-1}\right)$

$= \dfrac{2}{3}\cdot 27 + \dfrac{1}{9} - \dfrac{2}{3} - 1 = \dfrac{148}{9} = 16.4444$