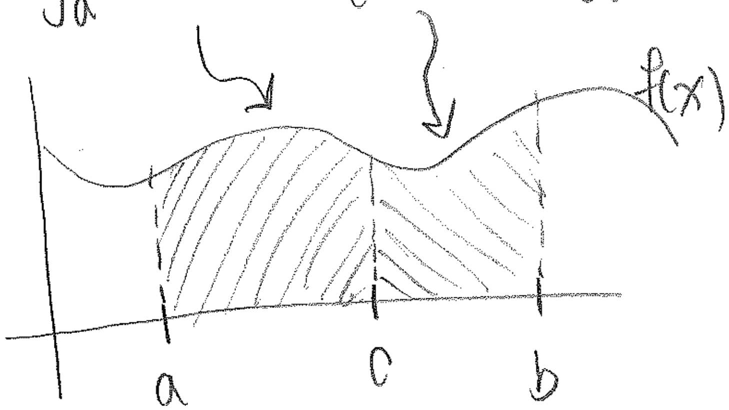
Jan. 16, 2013

Announcements:

- HW1 is due Friday
- WeBWork 5.2: Definite Integral is now due Fri (1/18)
- X-hour tomorrow (Thurs)

The Long Lost property (not really)

5. $\int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \int_a^b f(x)\,dx$



Example: $\int_0^4 f(x)\,dx = 5$, $\int_0^{12} f(x)\,dx = 11$

what does $\int_4^{12} f(x)\,dx = ?$

Property (5) gives us that:

$$\int_0^4 f(x)\,dx + \int_4^{12} f(x)\,dx = \int_0^{12} f(x)\,dx$$

$$5 + \int_4^{12} f(x)\,dx = 11 \Rightarrow \int_4^{12} f(x)\,dx = 6$$

Back to the Fundamental Theorem of Calculus

- The Area-so-far worksheet

FTC, part 1: If f is continous on $[a,b]$, then the function g defined by

$$g(x) = \int_0^t f(t)\,dt \quad a \le x \le b$$

is continuous on $[a,b]$ and differentiable on (a,b), and $g'(x) = f(x)$.

Examples: (1) $\frac{d}{dx}\int_4^x \sqrt{1+t^2}\,dt$

$$= \sqrt{1+x^2}$$

(2) $\frac{d}{dx}\int_1^x \sec\sqrt{t} \cdot \ln(\tan t)\,dt$

$$= \sec\sqrt{x} \cdot \ln(\tan x)$$

(3) $\frac{d}{dx}\int_x^3 \sqrt{t^2+1}\,dt$ (you have to switch the bounds first)

$$= \frac{d}{dx} - \left(\int_3^x \sqrt{t^2+1}\,dt\right) = -\sqrt{x^2+1}$$

FTC, part 2: If f is continuous on $[a,b]$, then $\int_a^b f(x)dx = F(b) - F(a)$

where F is any antiderivative of f, that is, a function s.t. $F' = f$.

Wait... that's all? Yup...

If $g(x) = \int_a^x f(x)dx$, we know (by part 1) that

$g'(x) = f(x)$, so g is an antiderivative of f.

watch FTC part 2 in action:

$$g(b) - g(a) = \int_a^b f(x)dx - \int_a^a f(x)dx$$

$$= \int_a^b f(x)dx \quad \text{YAY!}$$

Now we have a way to evaluate definite integrals that don't require limits! Let's try it:

<u>Examples</u>: (1) $\int_1^3 e^x dx$ $\qquad \frac{d}{dx} e^x = e^x$

$= e^3 - e^1$

(FTC says <u>an</u> antiderivative, we could use $e^x + 7$ or something but $(e^3 + 7) - (e^1 + 7) = e^3 - e^1$

(etc.)

(2) $\int_0^1 x^2 dx = \frac{x^3}{3}\Big|_0^1$ $\qquad \frac{d}{dx}\frac{x^3}{3} = x^2$

$= \frac{1}{3} - \frac{0}{3} = \frac{1}{3}$

explain this notation

(3) (practice)

$\int_0^{\pi/2} \cos x \, dx = \sin x \Big|_0^{\pi/2} = \sin(\pi/2) - \sin(0)$

$= 1$

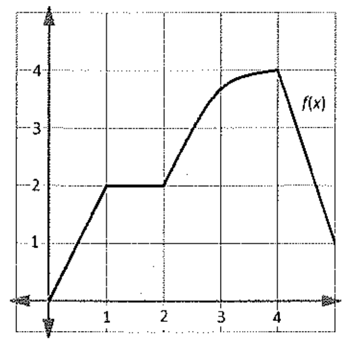
The short version: FTC

S'pose f is continuous on $[a,b]$

1. If $g(x) = \int_a^x f(t)\,dt$, then $g'(x) = f(x)$

2. $\int_a^b f(x)\,dx = F(b) - F(a)$, where $F' = f$.

The Area-So-Far and it's Derivative
(i.e. The Fundamental Theorem of Calculus)

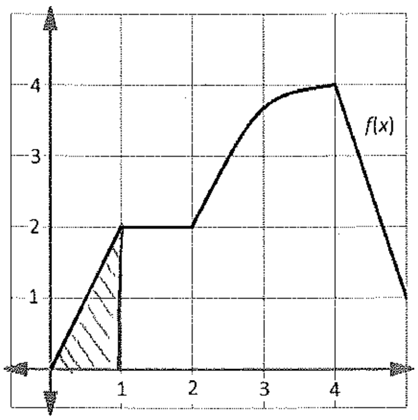


To the left is a graph of $f(x)$. We will be considering a special function which uses f. Let $g(x) = \int_0^x f(t)dt$. We will only be considering $g(x)$ for $x = 0$ to $x = 5$. Notice that $g(x)$ only depends on x.

1. What is $g(0)$ equal to?

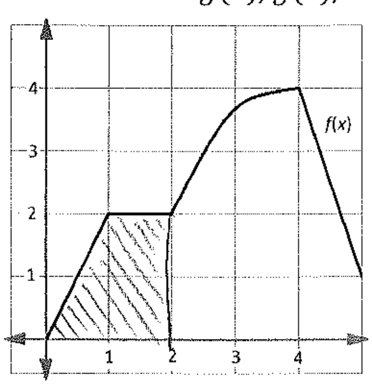
   $g(0) = \int_0^0 f(t)dt = 0$

2. Using the graph below, demonstrate what you would look at to determine $g(1)$. Now determine what $g(1)$ equals (approximately).

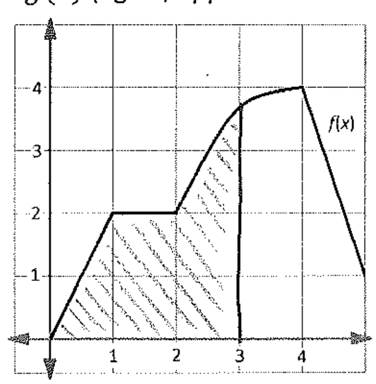


$g(1) = \int_0^1 f(t)dt \approx 1$

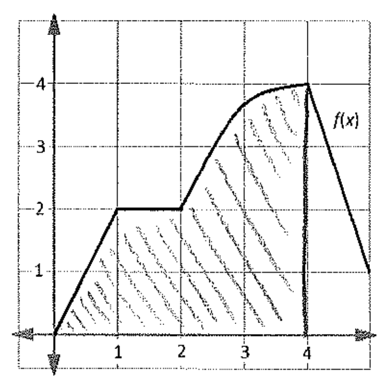
3. Do the same for $g(2)$, $g(3)$, and $g(4)$ (again, approximate their values).



$g(2) \approx 3$



$g(3) \approx 6$



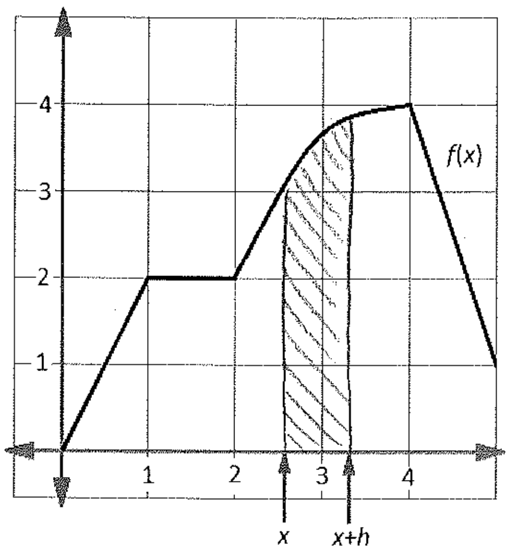
$g(4) \approx 9.75$

Functions like g can be thought of as the "area-so-far" function. We add up the area up to a certain x value. Now we would like to determine the derivative of g.

Recall: $g'(x) = \lim_{h \to 0} \frac{g(x+h)-g(x)}{h}$

We want to interpret this derivative pictorially.

4. What area does $g(x+h) - g(x)$ represent on the graph? Show in the figure below (x+h and x are marked in for you)



5. Given your answer to 4 above, what does $\frac{g(x+h)-g(x)}{h}$ appear to estimate? (it may help to think about what is happening to the picture of $g(x+h) - g(x)$ as h is getting smaller.)

If we think of the above slice as a rectangle, then h is the width, dividing $g(x+h)-g(x)$ by width (h) will give us the height of the rect. $\approx f(x)$

6. Conjecture what $g'(x) = \lim_{h \to 0} \frac{g(x+h)-g(x)}{h}$ is actually equal to.

$f(x)$