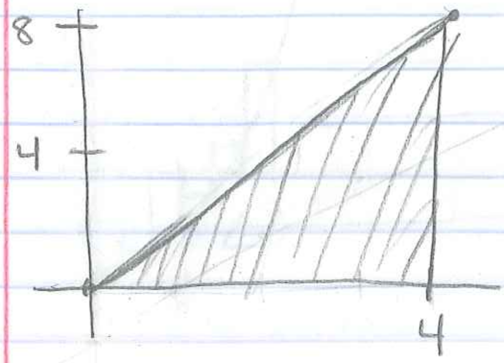
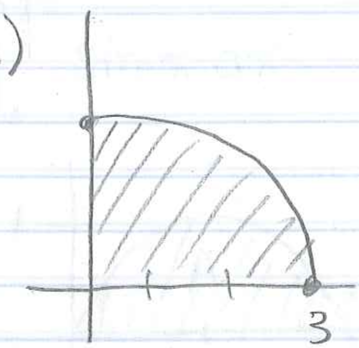
Jan. 14, 2013

Examples: (1) $\int_0^4 (2x)\,dx = \frac{1}{2}(4 \cdot 8) = 16$



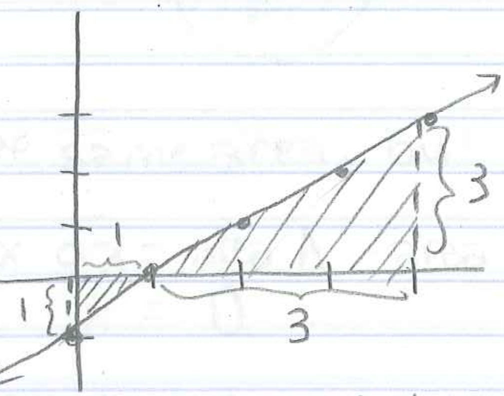
(2)



$\int_0^3 \sqrt{9 - x^2}\,dx$

$= \frac{9\pi}{4}$

(3)



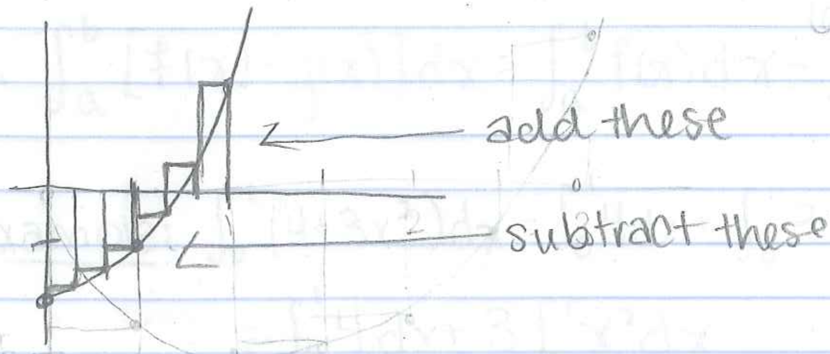
$\int_0^4 (x-1)\,dx$

* area under the x axis will be subtracted from area above the x-axis *

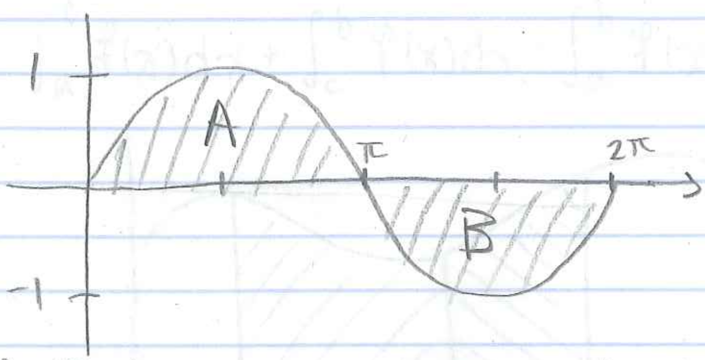
$\frac{1}{2}(3 \cdot 3) - \frac{1}{2}(1 \cdot 1) = \frac{9}{2} - \frac{1}{2} = \frac{8}{2} = 4$

This is already encoded in the integral, you only have to account for it when taking the integral using the basic geometry method.

$f(x) = x^2 - 2x$ on $[0,2]$ w/ 6 rect.



(4) $\int_0^{2\pi} \sin x \, dx$



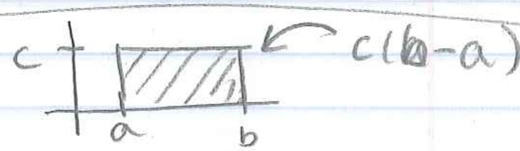
A, B have same area, but

$\int_0^{2\pi} \sin x \, dx = \text{area A} - \text{area B}$
$= 0$

Properties of Integrals

- $\int_b^a f(x)dx = -\int_a^b f(x)dx$ (b/c $\Delta x = \frac{a-b}{n}$ and $\frac{b-a}{n} = -\left(\frac{a-b}{n}\right)$)
- $\int_a^a f(x)dx = 0$

1. $\int_a^b c \, dx = c(b-a)$



2. $\int_a^b [f(x) + g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$

3. $\int_a^b c f(x)\,dx = c\int_a^b f(x)\,dx$ (c is a constant)

4. $\int_a^b [f(x) - g(x)]\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$

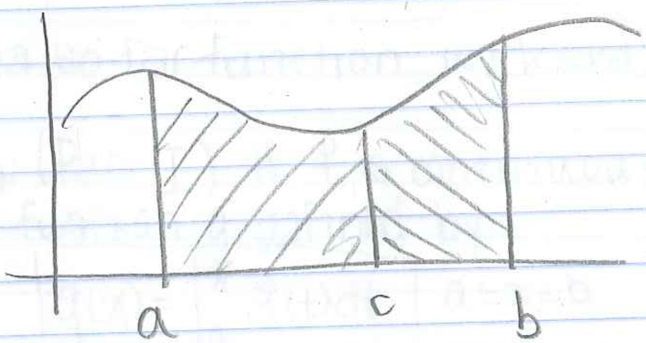
<u>Example</u>: $\int_0^1 (4+3x^2)\,dx = \int_0^1 4\,dx + \int_0^1 3x^2\,dx$

$= \int_0^1 4\,dx + 3\int_0^1 x^2\,dx$

$= 4 + 3\left(\frac{1}{3}\right) = 5$

* Use these rules to make integrals easier to solve *

5. $\int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \int_a^b f(x)\,dx$



<u>Example</u>: we know $\int_0^{11} f(x)\,dx = 7$, $\int_0^5 f(x)\,dx = 3$

What is $\int_5^{11} f(x)\,dx$?

$\underbrace{\int_0^5 f(x)\,dx}_{=3} + \int_5^{11} f(x)\,dx = \underbrace{\int_0^{11} f(x)\,dx}_{=7}$

$3 + \int_5^{11} f(x)\,dx = 7$

$\int_5^{11} f(x)\,dx = 4$

Comparison:

6. If $f(x) \geq 0$ for $a \leq x \leq b$, then $\int_a^b f(x)\,dx \geq 0$

7. If $f(x) \geq g(x)$ for $a \leq x \leq b$, then $\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$