# Math 2 - HW 1

5.1: 2, 4, 14, 24
5.2: 20, 34, 36, 48, 50, 52

5.1

2. (a) (i) $L_6 = 9\cdot 2 + 8.75\cdot 2 + 8.25\cdot 2 + 7.25\cdot 2 + 6\cdot 2 + 4\cdot 2$
$= 88.1$

(ii) $R_6 = 8.75\cdot 2 + 8.25\cdot 2 + 7.25\cdot 2 + 6\cdot 2 + 4\cdot 2 + 1\cdot 2$
$= 72.1$

5 pts

(iii) $M_6 = 9\cdot 2 + 8.5\cdot 2 + 7.75\cdot 2 + 6.5\cdot 2 + 5\cdot 2 + 2.5\cdot 2$
$= 78.5$

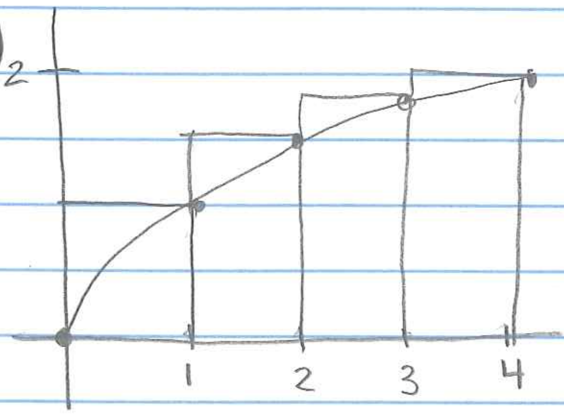
(b) overestimate

(c) underestimate

(d) $M_6$, the midpt overestimates less than $L_6$ and underestimates less than $R_6$.

4. (a)

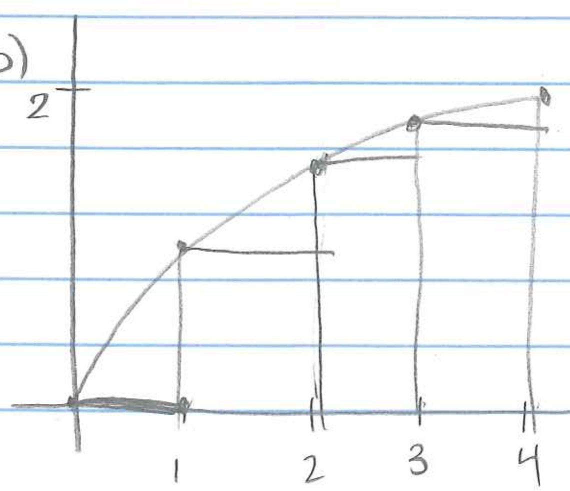
3 pts




$R_4 = \sqrt{1}\cdot 1 + \sqrt{2}\cdot 1 + \sqrt{3}\cdot 1 + \sqrt{4}\cdot 1$
$= \boxed{6.14626}$

overestimate

(b)




$L_4 = \sqrt{0}\cdot 1 + \sqrt{1}\cdot 1 + \sqrt{2}\cdot 1 + \sqrt{3}\cdot 1$
$= \boxed{4.14626}$

underestimate

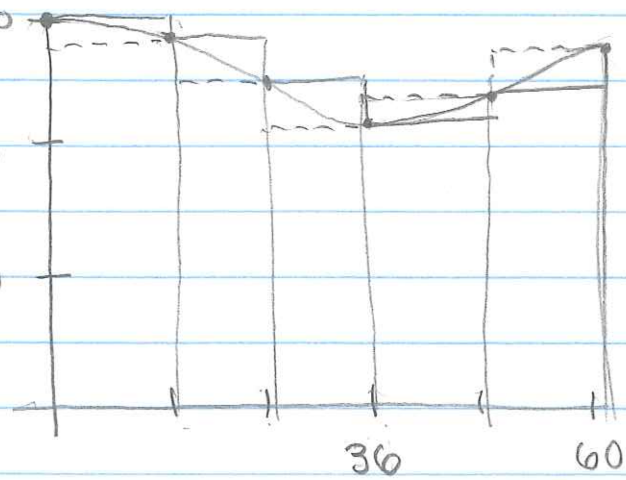
14. (a) length of interval: 12 s

Estimate: $(30\ m/s)(12s) + (28\ m/s)(12\ s) + (25\ m/s)(12s) + (22\ m/s)(12s)$
$+ (24\ m/s)(12s)$

$= 1548\ m$

4 pts (b) $(28\ m/s)(12s) + (25\ m/s)(12\ s) + (22\ m/s)(12s) + (24\ m/s)(12s) + (27\ m/s)(12s)$
$= 1512$

(c)

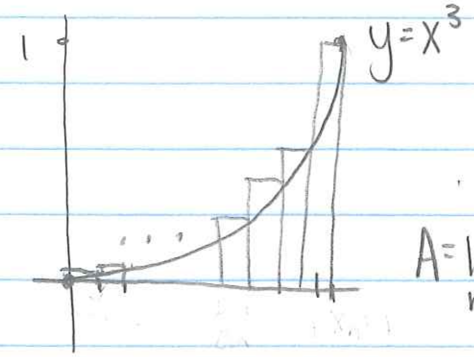



(a) upper
(b) lower

OR

ambiguous, since both have rectangles that overestimate and underestimate.

24. (a)




$\Delta x = \frac{1-0}{n} = \frac{1}{n}$

$x_1 = \frac{1}{n}, x_2 = \frac{2}{n}, x_3 = \frac{3}{n}, \ldots, x_n = \frac{n}{n} = 1$

3 pts

$A = \lim_{n\to\infty} R_n = \boxed{\lim_{n\to\infty} \left[\left(\frac{1}{n}\right)^3 \frac{1}{n} + \left(\frac{2}{n}\right)^3\left(\frac{1}{n}\right) + \left(\frac{3}{n}\right)^3\left(\frac{1}{n}\right) + \ldots + (1)^3\left(\frac{1}{n}\right)\right]}$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^3 \frac{1}{n}$

(b) $A = \lim_{n\to\infty} \left[\frac{1}{n^4}\left(1^3 + 2^3 + 3^3 + \ldots + n^3\right)\right] = \lim_{n\to\infty} \left[\frac{1}{n^4}\left(\frac{n(n+1)}{2}\right)^2\right]$

$= \lim_{n\to\infty} \frac{(n+1)^2}{4n^2} = \lim_{n\to\infty} \frac{n^2+2n+1}{4n^2} = \lim_{n\to\infty}\left[\frac{1}{4} + \frac{1}{n} + \frac{1}{4n^2}\right]$

$= \boxed{\frac{1}{4}}$

5.2

20. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{x_i^*}{(x_i^*)^2+4} \Delta x$

3pts

$$\boxed{\int_1^3 \frac{x}{x^2+4}\,dx}$$

34. (a) $\int_0^2 g(x)\,dx = \frac{1}{2}(2\cdot 4) = 4$

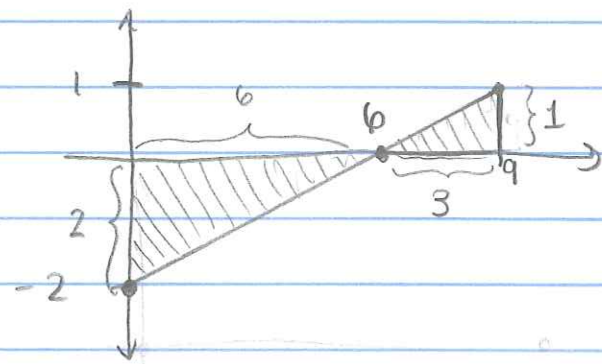
3pts (b) $\int_2^6 g(x)\,dx = \frac{1}{2}(\pi(2)^2) = -2\pi$

(c) $\int_0^7 g(x)\,dx = \int_0^2 g(x)\,dx + \int_2^6 g(x)\,dx + \int_6^7 g(x)\,dx = 4.5 - 2\pi$

36. $\int_0^9 \left(\frac{1}{3}x - 2\right) dx$

$= \frac{1}{2}(1\cdot 3) - \frac{1}{2}(2\cdot 6)$

3pts $= \frac{3}{2} - 6 = \boxed{\frac{-9}{2}}$




48. $\int_1^4 f(x)\,dx = \int_1^5 f(x)\,dx - \int_4^5 f(x)\,dx$

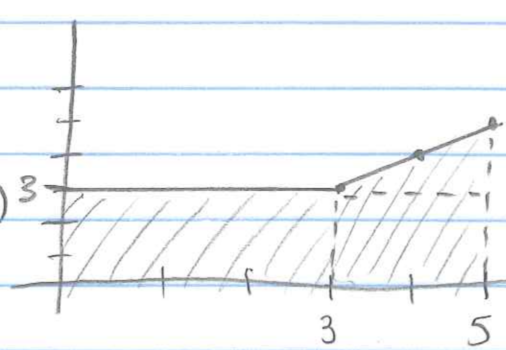
3 pts

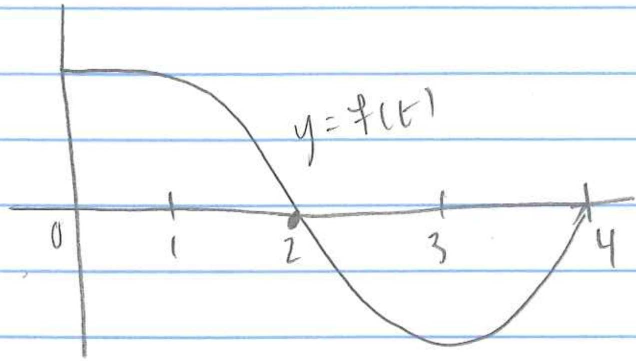
$= 12 - 3.6 = \boxed{8.4}$

50. $f(x) = \begin{cases} 3 & \text{for } x<3 \\ x & \text{for } x \geq 3 \end{cases}$

3 pts $\int_0^5 f(x)\,dx = 3\cdot 3 + 2\cdot 3 + \frac{1}{2}(2\cdot 2)$

$= 9+6+2 = \boxed{17}$



52. 


$F(x) = \int_2^x F(t)\,dt$

3 pts

$F(0) = \int_2^0 f(t)\,dt \le 0$ } switched bounds

$F(1) = \int_2^1 f(t)\,dt \le 0$

$F(2) = \int_2^2 f(t)\,dt = 0$

$F(3) = \int_2^3 f(t)\,dt \le 0$ } area under the x-axis

$F(4) = \int_2^4 f(t)\,dt \le 0$