(2) $\int t^2 e^t \, dt$ $\quad u = t^2 \quad dv = e^t dt$

$du = 2t\,dt \quad v = e^t$

$= t^2 e^t - \int 2te^t \, dt$ Still can't take the integral. Use integration by parts again!

$u = 2t \quad dv = e^t dt$

$du = 2dt \quad v = e^t$

$= t^2 e^t - \left[2te^t - \int 2e^t \, dt \right]$

$= \boxed{t^2 e^t - 2te^t + 2e^t + C}$

(3) $\int e^x \sin x \, dx \quad u = e^x \quad dv = \sin x \, dx$

$du = e^x dx \quad v = -\cos x$

$= -e^x \cos x - \int -e^x \cos x \, dx$

$= -e^x \cos x + \int e^x \cos x \, dx \quad u = e^x \quad dv = \cos x \, dx$

$du = e^x dx \quad v = \sin x$

$= -e^x \cos x + e^x \sin x - \int e^x \sin x \, dx$

Where are we at?

$\int e^x \sin x \, dx = -e^x \cos x + e^x \sin x - \int e^x \sin x \, dx$

Solve for the integral.

$2 \int e^x \sin x \, dx = -e^x \cos x + e^x \sin x$

$\int e^x \sin x \, dx = \boxed{\frac{-e^x \cos x + e^x \sin x}{2} + C}$

Integration by Parts + Definite Integrals

$$\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$$

* at the end, plug in the bounds like you always would

Examples:

(1) $\int_1^2 x \ln x\,dx$ $\quad u = \ln x \quad dv = x\,dx$ $\quad du = \frac{1}{x}dx \quad v = \frac{x^2}{2}$

$$= \ln x \cdot \frac{x^2}{2}\Big|_1^2 - \int_1^2 \frac{x^2}{2}\cdot\frac{1}{x}\,dx$$

$$= \ln 2 \cdot \frac{(2)^2}{2} - \int_1^2 \frac{x}{2}\,dx$$

$$= \ln 2 \cdot 2 - \left(\frac{x^2}{4}\Big|_1^2\right) = 2\ln 2 - 1 + \frac{1}{4}$$

(2) (harder) $\int_0^1 \tan^{-1} x\,dx$ $\quad u = \tan^{-1} x \quad dv = 1\,dx$ $\quad du = \frac{1}{1+x^2}dx \quad v = x$

$$= \tan^{-1} x \cdot x\Big|_0^1 - \int_0^1 \frac{x}{1+x^2}\,dx$$

use substitution

$w = 1 + x^2$

$dw = 2x\,dx$

$$= \tan^{-1} 1 - \int_{x=0}^{x=1} \frac{1}{2}\frac{1}{w}\,dw$$

$$= \frac{\pi}{4} - \frac{1}{2}\ln w\Big|_{x=0}^{x=1} = \frac{\pi}{4} - \frac{1}{2}\ln(1+x^2)\Big|_0^1 = \frac{\pi}{4} - \frac{\ln 2}{2}$$

Integration by Parts

Use integration by parts (in conjuction with other integration techniques you already know) to solve the following integrals:

1. $\int_0^{\pi/6} x \cdot \cos(3x)\, dx$

2. $\int_1^2 x^2 \cdot \ln x\, dx$

3. $\int_0^{\pi/2} (x^2 + 2x) \cdot \cos x\, dx$

4. $\int_0^1 \tan^{-1} x\, dx$

5. $\int \cos x \cdot \ln(\sin x)\, dx$