Feb. 6, 2013
Announcements:

Integration by Parts

Find $\int x \sin x \, dx$. We are powerless w/ only u-sub. Let's get a new technique!

Recall:
Chain Rule $\longleftrightarrow$ Substitution

Now:
Product Rule $\longleftrightarrow$ Integration by parts

Deriving Integration by Parts:

$$\frac{d}{dx}\left[f(x)g(x)\right] = g(x)f'(x) + f(x)g'(x)$$

Now take integral of both sides:

$$f(x)g(x) = \underbrace{\int g(x)f'(x)\,dx}_{\text{subtract this term from both sides}} + \int f(x)g'(x)\,dx$$

$$\boxed{f(x)g(x) - \int g(x)f'(x)\,dx = \int f(x)g'(x)\,dx}$$

The easier to remember form:

$$\boxed{\int u\,dv = uv - \int v\,du}$$

$u = f(x)$
$du = f'(x)\,dx$
$v = g(x)$
$dv = g'(x)\,dx$

Back to our dilemma:

$$\int x \sin x \, dx$$

$u = x$
$du = dx$
$dv = \sin x \, dx$
$v = -\cos x$

*you want to assign $f(x), g(x)$ so you make the integral you have to take simpler.

$\int x \sin x \, dx = x(-\cos x) - \int 1(-\cos x)\,dx$

$= -x\cos x - \int(-\cos x)\,dx$

$= -x\cos x + \sin x + C$

Did that work?

$\frac{d}{dx}(-x\cos x + \sin x + C)$

$= (-1)\cos x + (-x)(-\sin x) + \cos x$

$= x\sin x$

* you want to set u, dv such that you end up with an easier integral*

The Not Helpful Way

$u = \sin x \qquad dv = x\,dx$

$du = \cos x\,dx \qquad v = \frac{x^2}{2}$

$$\frac{x^2}{2}\sin x - \underbrace{\int \frac{x^2}{2}\cos x\,dx}_{\text{this integral is } \underline{\text{not}} \text{ easier.}}$$

Examples:

(1) $\int \ln x\,dx \qquad u = \ln x \qquad dv = 1\,dx$

$du = \frac{1}{x}dx \qquad v = x$

$= x\ln x - \int x \cdot \frac{1}{x}\,dx = x\ln x - \int 1\,dx$

$= \boxed{x\ln x - x + C}$

(2) $\int t^2 e^t \, dt$ $\quad u = t^2 \quad dv = e^t dt$

$du = 2t\,dt \quad v = e^t$

$= t^2 e^t - \int 2te^t \, dt$ Still can't take the integral. Use integration by parts again!

$u = 2t \quad dv = e^t dt$

$du = 2dt \quad v = e^t$

$= t^2e^t - \left[2te^t - \int 2e^t \, dt\right]$

$= \boxed{t^2e^t - 2te^t + 2e^t + C}$

(3) $\int e^x \sin x \, dx$ $\quad u = e^x \quad dv = \sin x \, dy$

$du = e^x dx \quad v = -\cos x$

$= -e^x \cos x - \int -e^x \cos x \, dx$

$= -e^x \cos x + \int e^x \cos x \, dx$ $\quad u = e^x \quad dv = \cos x \, dx$

$du = e^x dx \quad v = \sin x$

$= -e^x \cos x + e^x \sin x - \int e^x \sin x \, dx$

Where are we at?

$$\int e^x \sin x \, dx = -e^x \cos x + e^x \sin x - \int e^x \sin x \, dx$$

Solve for the integral.

$$2\int e^x \sin x \, dx = -e^x \cos x + e^x \sin x$$

$$\int e^x \sin x \, dx = \boxed{\frac{-e^x \cos x + e^x \sin x}{2} + C}$$