Feb. 4, 2013

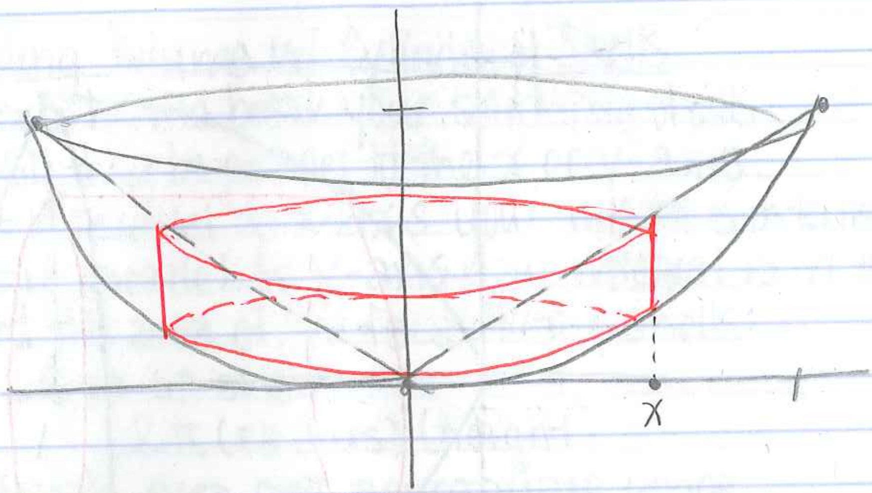
Announcements: WeBWork: 6.3 Due Wed | No Class Fri

6.5 Due Fri | X-hour this week.

HW4 Due Mon.

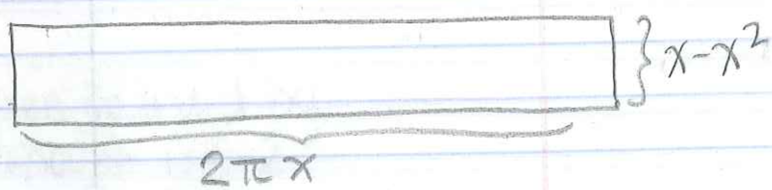
Let's do some examples we are familiar with:

(2) Find the volume of the solid obtained by rotating about the y-axis the region between $y=x$ and $y=x^2$



radius: x

height: $x-x^2$



Area of shell:

$$2\pi x(x-x^2)$$

$$=2\pi x^2-2\pi x^3$$

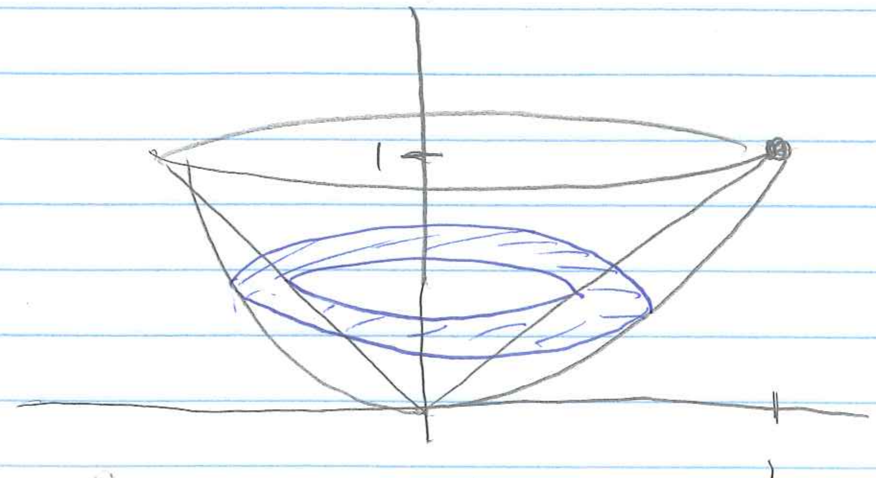
$$\int_0^1 (2\pi x^2-2\pi x^3)\,dx = \frac{2\pi x^3}{3}-\frac{2\pi x^4}{4}\Big|_0^1$$

$$=\frac{2\pi}{3}-\frac{2\pi}{4}=\frac{4\pi-3\pi}{6}$$

$$=\boxed{\pi/6}$$

The washer method

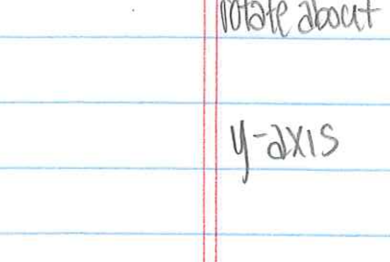
- Volume of the solid obtained by rotating about the y-axis the region between $y=x$ and $y=x^2$.



Integrate with respect to y

$x=y,\ x=\sqrt{y}$

Area of washer: $\pi(\sqrt{y})^2 - \pi(y^2)$



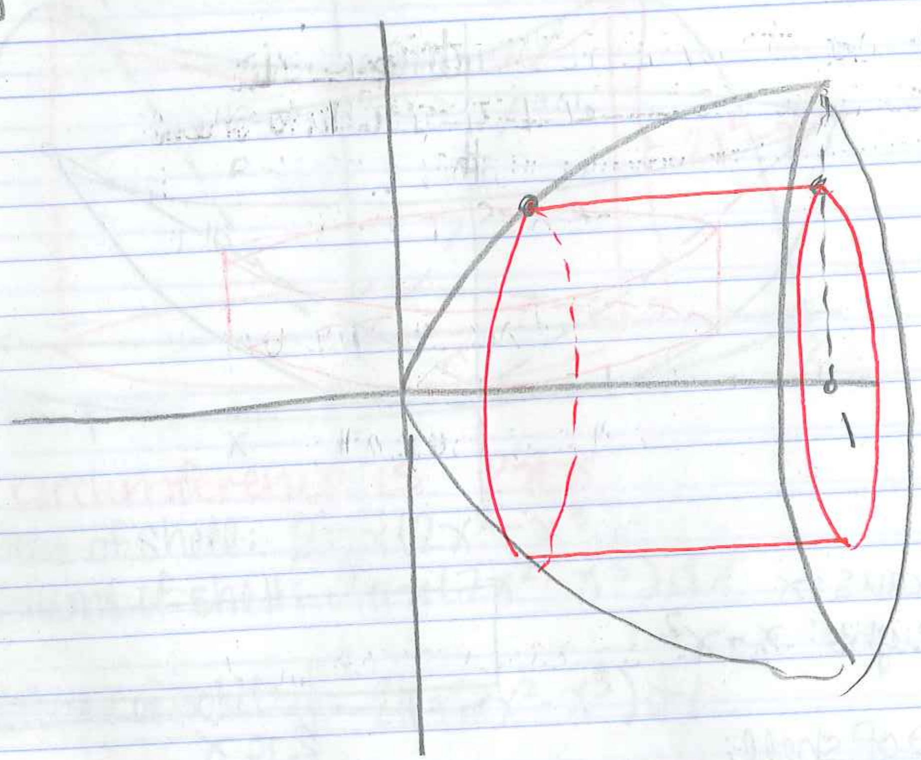
$$\text{Volume} = \int_0^1 \pi y - \pi y^2 \, dy = \frac{\pi}{2}y^2 - \frac{\pi}{3}y^3 \Big|_0^1$$

$$= \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$$

Recap:

rotate about	Disk/Washer	Cylindrical Shells
y-axis	Volume $= \int^d A(y)\,dy$ $A(y)$ area of cross-section $A(y) = \pi(\text{outer radius})^2 - \pi(\text{inner radius})^2$	Vol $= \int_a^b 2\pi x f(x)\,dx$ $f(x)$ = height of shell
x-axis	Volume $= \int_a^b A(x)\,dx$ $A(x)$ is area of a cross-section $A(x) = \pi(\text{outer radius})^2 - \pi(\text{inner radius})^2$	Vol $= \int_c^d 2\pi y f(y)\,dy$ $f(y)$ = width of shell

(3) Use cylindrical shells to find the volume of the solid obtained by rotating about the x-axis the region under the curve $y=\sqrt{x}$ from 0 to 1.



height: distance from $\sqrt{x}=y$ to $x=1$
b/c the distance is horizontal we need this in terms of y.

so $x=y^2$

distance from y-axis to $x=y^2$ is y^2

so distance from $\sqrt{x}=y$ to $x=1$

is $\underline{1-y^2}$

<u>Circumference</u>: $2\pi y$

<u>Area of shell</u>: $2\pi y(1-y^2)$

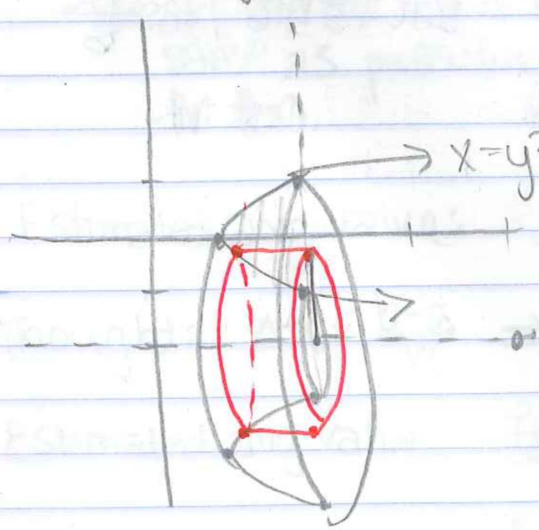
Volume: $\int_0^1 2\pi y(1-y^2)\,dy = \frac{2\pi y^2}{2} - \frac{2\pi y^4}{4}\Big|_0^1$

$= \pi - \frac{\pi}{2} = \boxed{\frac{\pi}{2}}$

Finding Volume w/ Cylindrical Shells

- Decide how to break your solid into shells
  Will they be parallel to the x or y-axis.
  - If parallel to x-axis your integral is in terms of y
  - If parallel to y-axis your integral is in terms of x
- Find the area of the face of the shells:
  Area of shell =
  $2\pi(\text{radius})(\text{height})$
- Integrate Area over appropriate range.

Example: volume of region bounded by
$x=y^2+1$, $x=2$ about $y=-2$

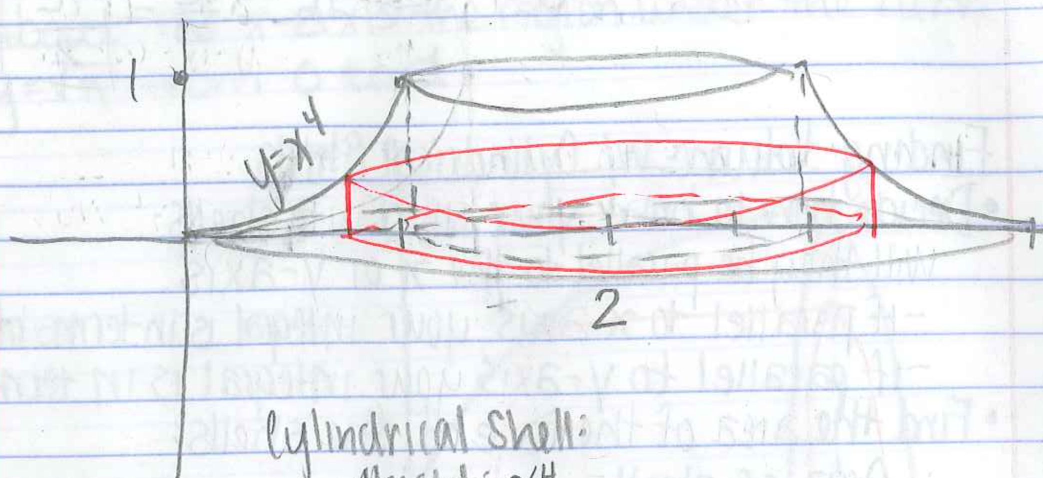


Area of shell:
height: $2-y^2-1$
radius: $1+y$

$\int_{-1}^{1} 2\pi(1+y)(2-y^2-1)\,dy$

region bounded by

Example: $y=x^4$, $y=0$, $x=1$
rotated about $x=2$



Cylindrical Shell:
Height: x^4
Radius: $2-x$

$$\int_0^1 2\pi(2-x)x^4\,dx$$

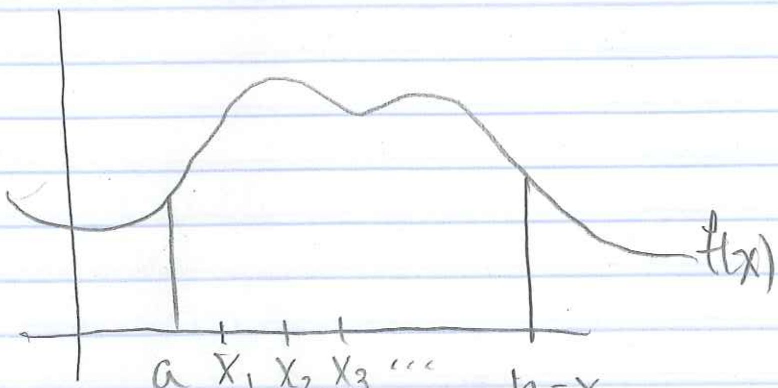
$$= 2\pi\int_0^1 2x^4 - x^5\,dx = 2\pi\left(\frac{2x^5}{5} - \frac{x^6}{6}\right)\Big|_0^1$$

$$= 2\pi\left(\frac{2}{5} - \frac{1}{6}\right) = \frac{7\pi}{6}$$

Average Value of a Function

We can find avg value when we have a discrete set of points, but what do we do with a continuous function?

Say you want to know the average temp for a day. We need some machinery to make this happen.



avg value between a and b:

Let's estimate first using Riemann Sums.
Break interval into n rectangles. This is the same as pretending like we only have n values for f(x).

Estimated avg value: $\dfrac{f(x_1)+f(x_2)+\dots+f(x_n)}{n}$

Thought: $\Delta x = \dfrac{b-a}{n} \Rightarrow n = \dfrac{b-a}{\Delta x}$

Estimated avg value: $\dfrac{f(x_1)+f(x_2)+\dots+f(x_n)}{(b-a)/\Delta x}$

$$= \frac{f(x_1)\Delta x + f(x_2)\Delta x + \dots + f(x_n)\Delta x}{(b-a)}$$

$$= \frac{R_n}{b-a}$$

For average values:

$$\lim_{n\to\infty} \frac{R_n}{b-a} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

DEF: The average value of a function f on the interval $[a,b]$ is

$$f_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

Example: Avg value of $f(x) = 1+x^2$ on the interval $[-1,2]$.

$$f_{avg} = \frac{1}{2-(-1)}\int_{-1}^{2} 1+x^2\,dx$$

$$= \frac{1}{3}\left(x+\frac{x^3}{3}\right)\Big|_{-1}^{2} = \frac{1}{3}\left(2+\frac{8}{3}\right) - \frac{1}{3}\left(-1-\frac{1}{3}\right)$$

$$= \frac{6}{9}+\frac{8}{9}+\frac{4}{9} = 2$$

Ex: A particle has velocity function $v(t) = t\sin(t^2)$ on the interval $[0,2]$. What is it's avg velocity on this interval?

$$V_{avg} = \frac{1}{2-0}\int_0^2 t\sin(t^2)\,dt \qquad u=t^2 \quad du = 2t\,dt$$

$$= \frac{1}{2}\int_{t=0}^{t=2} \frac{1}{2}\sin u\,du$$

$$= \frac{1}{2}\left(-\frac{1}{2}\cos u\right)\Big|_{t=0}^{t=2} = -\frac{1}{4}\cos t^2\Big|_0^2$$

$$= -\frac{1}{4}\cos 4 + \frac{1}{4}$$